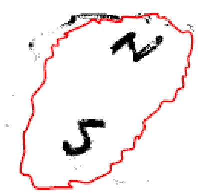
Magnetic field:

Until 1820, the only magnetism known was that of iron magnets and of "lodestones natural magnets of iron-rich ore.

2000 years ago: Greeks were aware of "Magnetite" stones attract pieces of iron

natural magnets

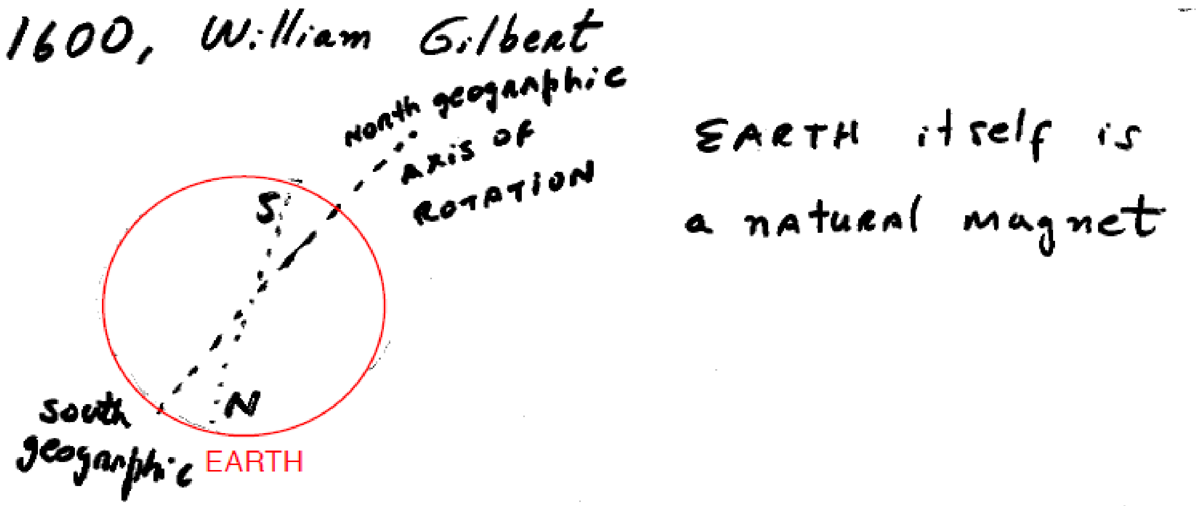
Every magnetic has two pole. Needle 



Individual magnetic charges (called magnetic monopoles) are predicted by certain theories, their existence has not been confirmed.

Thers is so far no conclusive evidance of the existance of isolated magnetic poles

Magnetic poles always occur in pairs



There are two ways to produce magnetic field:

1. One way is to use moving electrically charged particles, such as a current in a wire, to make an **electromagnet.** The current produces a magnetic field that can be used, for example, to control a computer hard drive or to sort scrap metal.
2. The other way to produce a magnetic field is by means of elementary particles such as electrons because these particles have an *intrinsic* magnetic field around them.

The magnetic filed B:

The force on an electric charge *q* depends on

1. Not only the position of charge that where it is : (r)
2. It also depends on how is it moving (V)

So a charged particle fired or accelerated in magnetic field $\vec{B}$, into various directions and speeds for the particle and determining the force $\vec{F}_B$ that acts on the particle at that point.

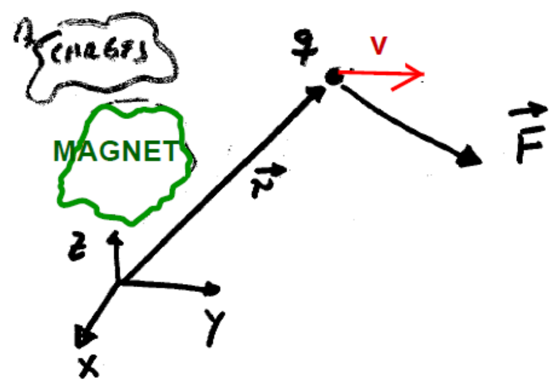
It was observed that

$\vec{F}_B = 0$, when the particle's velocity is along a particular axis through the point, force is zero.

For all other directions of $\vec{V}$, the magnitude of $\vec{F}_B$ is always proportional to $v \sin\theta$, where θ is the angle between the zero-force axis and the direction of $\vec{V}$.

Furthermore, the direction of $\vec{F}_B$ is always perpendicular to the direction of $\vec{V}$.

These results suggest that a cross product is involved.



$$\vec{F} = \underbrace{q\,\vec{E}(\vec{r})}_{\text{Force independent of the motion of the charge}} + \underbrace{q\,\vec{v} \times \vec{B}(\vec{r})}_{\text{Force that depend on the velocity } \vec{v} \text{ of the charge.}}$$

Magnitude of the force acting on a particle in a magnetic field is proportional to the charge *q* and speed *v* of the particle.

i. F = 0, if q = 0, V = 0
ii. F = 0. If V and B are parallel or ant parallel.
iii. F = maximum, when V and B are perpendicular
iv. If q is positive, then the force has the same sign as and thus must be in the same direction; that is, for positive q, force is directed along the thumb
v. If q is negative, then the force and cross product V×B has the opposite sign and thus must be in the opposite direction; that is, for negative q, force is directed opposite to the thumb.
vi. F_B, never has a component parallel to $\vec{V}$ .This means that F_B cannot change the particle's speed v (and thus it cannot change the particle's kinetic energy). The force can change only the direction of $\vec{V}$; only in this sense F_B can accelerate the particle.

The mass spectrometer

Mass spectrometer, can be used to measure the mass of an ion; an ion of mass *m* (to be measured) and charge *q* is produced in source *S*.

The initially stationary ion is accelerated by the electric field due to a potential difference *V*.

The ion leaves *S* and enters a separator chamber in which a uniform magnetic field is perpendicular to the path of the ion.

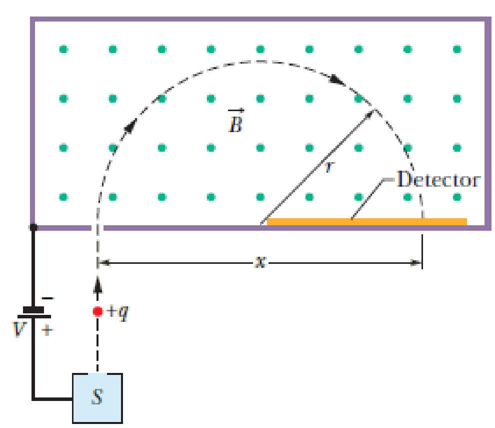
A wide detector lines the bottom wall of the chamber, and the causes the ion to move in a semicircle and thus strike the detector.

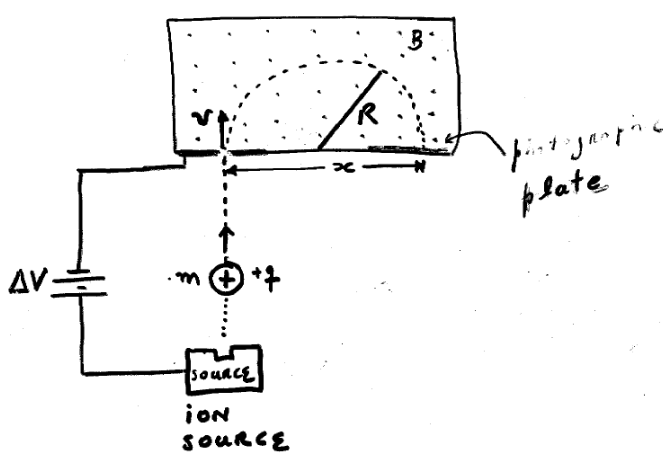
Because the (uniform) magnetic field causes the (charged) ion to follow a circular path, we can relate the ion's mass *m* to the path's radius *r* with (r = mv / |q| B).

From Fig., we see that r = x/2 (the radius is half the diameter).

However, we lack the ion's speed *v* in the magnetic field after the ion has been accelerated due to the potential difference V.

To relate *v* and *V*, we use the fact that mechanical energy (E_{mec} = K + U) is conserved during the acceleration.





$$q\,\Delta V = \frac{1}{2} m v^2 \qquad \frac{m v^2}{R} = q v B$$

$$v = \sqrt{\frac{2 q \Delta V}{m}} \qquad R = \frac{m v}{q B} = \frac{x}{2}$$

$$x = 2 \frac{m}{q B} v$$

$$= 2 \frac{m}{q B} \sqrt{\frac{2 q \Delta V}{m}} = \sqrt{\frac{8 m \Delta V}{q B^2}}$$

$$\boxed{\frac{m}{q} = \frac{B^2}{8 \Delta V} x^2}$$

Isotopes of different mass m (same q) will strike the photographic plate at different values of x.

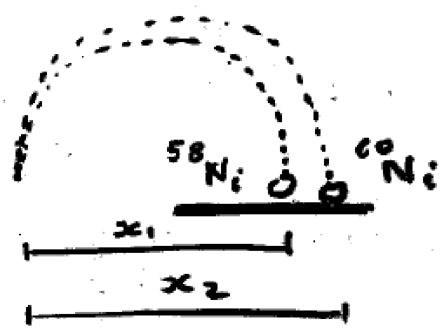
Example A ^{58}Ni ion of charge $+e$ and mass $m = 9.62 \times 10^{-26}$ kg is accelerated through a potential difference of 3 kV and deflected in a magnetic field of 0.12 T

a) Find the radius of curvature of the orbit of the ion.

$$\frac{9.62 \times 10^{-26}\,\text{kg}}{1.6 \times 10^{-19}\,\text{C}} = \frac{(0.12\,\text{T})^2}{8 \times 3 \times 10^3\,\text{V}} x^2 \Rightarrow \begin{array}{l} x = 1.0020\,\text{m} \\ R = 0.5010\,\text{m} \end{array}$$

b) Find the difference in the radii of curvature of ^{58}Ni and ^{60}Ni

$$\frac{m_1}{m_2} = \frac{x_1^2}{x_2^2}$$



we take $\frac{m_1}{m_2} = \frac{58}{60}$

$$\Rightarrow \frac{x_1}{x_2} = 0.983 = \frac{R_1}{R_2}$$

Since $R_1 = 0.5010$ m

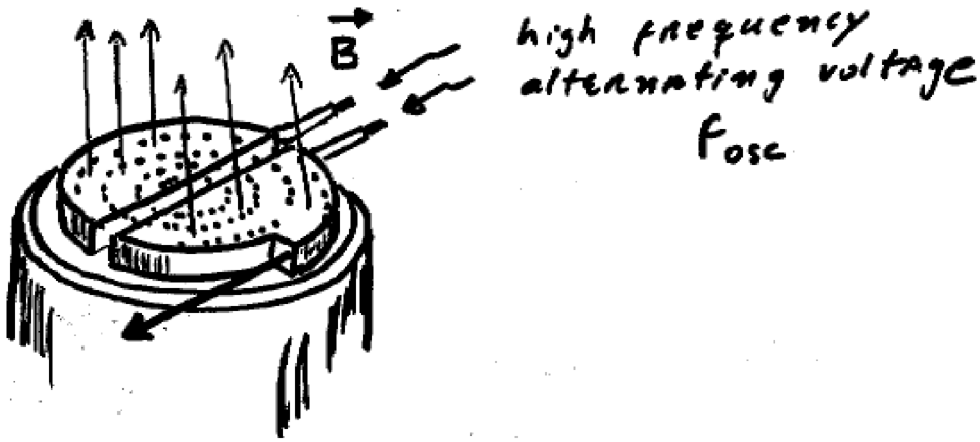
then $R_2 = 0.5095\,\text{m} \Rightarrow R_2 - R_1 = 8.56$ mm

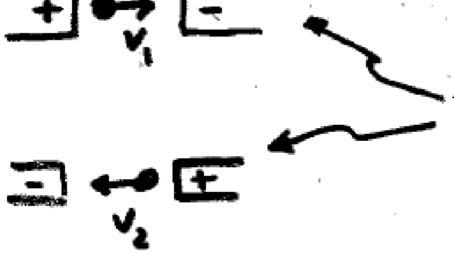
Cyclotrons and Synchrotrons

- Beams of high-energy particles, such as high-energy electrons and protons, have been enormously useful in probing atoms and nuclei to reveal the fundamental structure of matter.
- Such beams were instrumental in the discovery that atomic nuclei consist of protons and neutrons and in the discovery that protons and neutrons consist of quarks and gluons.
- The challenge of such beams is how to make and control them.
- Because electrons and protons are charged, they can be accelerated to the required high energy if they move through large potential differences.
- Because electrons have low mass, accelerating them in this way can be done in a reasonable distance.
- However, because protons (and other charged particles) have greater mass, the distance required for the acceleration is too long.

Solution to this problem is first to let protons and other massive particles move through a modest potential difference (so that they gain a modest amount of energy) and then use a magnetic field to cause them to circle back and move through a modest potential difference again.

If this procedure is repeated thousands of times, the particles end up with a very large energy. Here we discuss two accelerators that employ a magnetic field to repeatedly bring particles back to an accelerating region, where they gain more and more energy until they finally emerge as a high-energy beam.





the polarity of the electrodes is changed in a synchronized way with the motion of the particle

We already know that, given a particle of mass m and charge q, it will circle inside a uniform magnetic field with a frequency $f = \frac{1}{2\pi} \frac{q}{m} B$ regardless of the particle's speed

So, in a cyclotron the alternating voltage (see previous figure) is tuned until $f_{osc} = \frac{1}{2\pi}\frac{q}{m}B$

<u>Example</u> A cyclotron uses a magnetic field B = 1.5 T and it is going to be used to accelerate protons.

a) What should be the frequency of the alternating voltage?

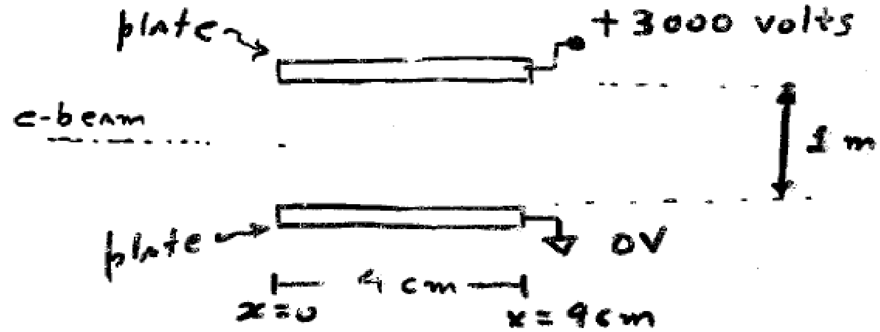
$$f_{osc} = \frac{1}{2\pi}\frac{1.6\times10^{-19}\,C \times 1.5\,T}{1.67\times10^{-27}\,kg} = 23\ MHz$$

b) If the radius of the D-shape electrodes is 0.5 m, what is the kinetic energy of the protons when they emerge?

Practice:
Read sample problem 29-5

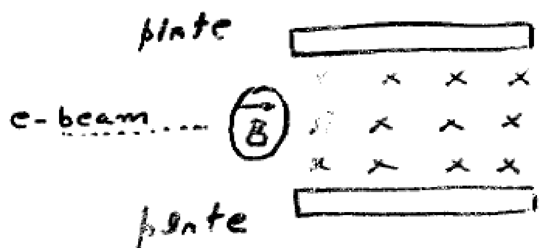
Practice problems: 16E, 19E, 20E, 21E
(page 683) 22P, 28P

EXERCISE: Draw schematically the trajectory followed by the indicated electron beam

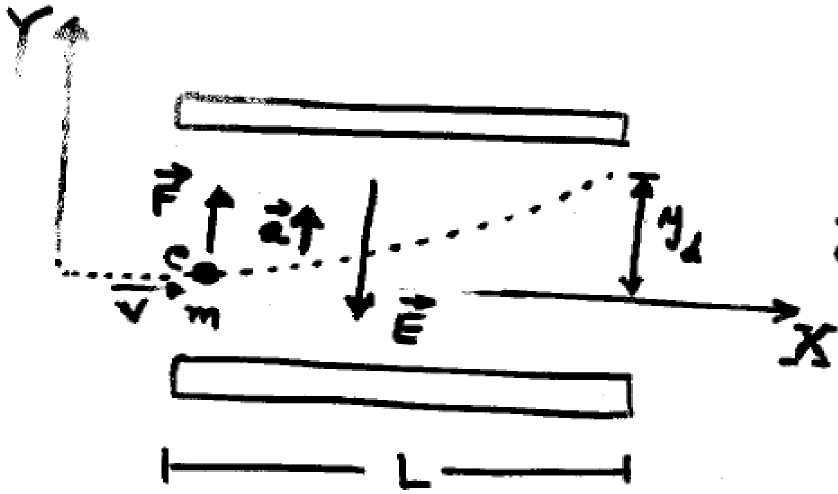


Find the position of the beam when it exists the plates (at $x = 4\,cm$)

EXERCISE: Draw schematically the trajectory followed by the electron beam



(No electric field are applied in this case.)



$$\vec{F} = e\,\vec{E}$$

$$\vec{a} = \frac{\vec{F}}{m} = \frac{e}{m}$$

$$y = \frac{1}{2} a t^2$$

How much does the electron deflect after passing the plates? $y_d = ?$

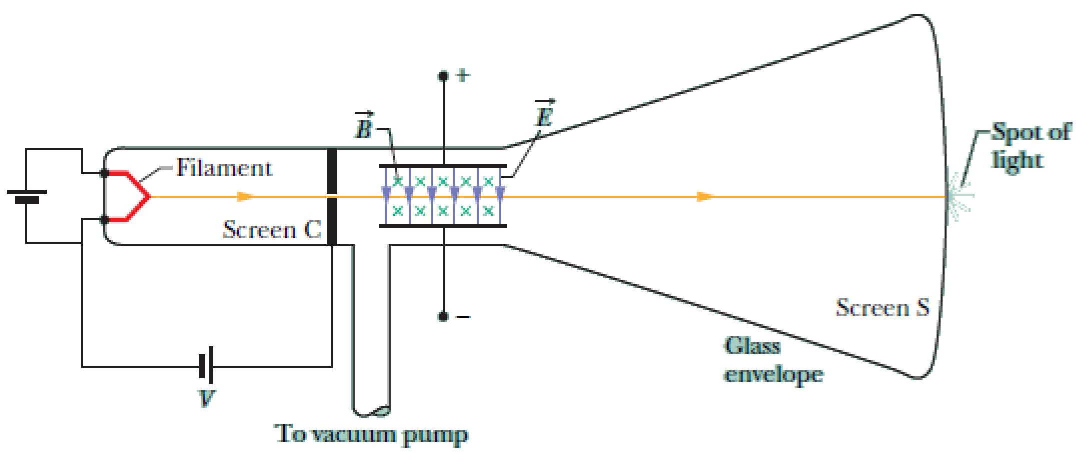
Answer: We obtain y_d when $t = \frac{L}{v}$

$$y_d = \frac{1}{2} a \left(\frac{L}{v}\right)^2$$

$$\boxed{y_d = \frac{1}{2} \frac{e}{m} E \frac{L^2}{v^2}}$$

Discovery of the Electron

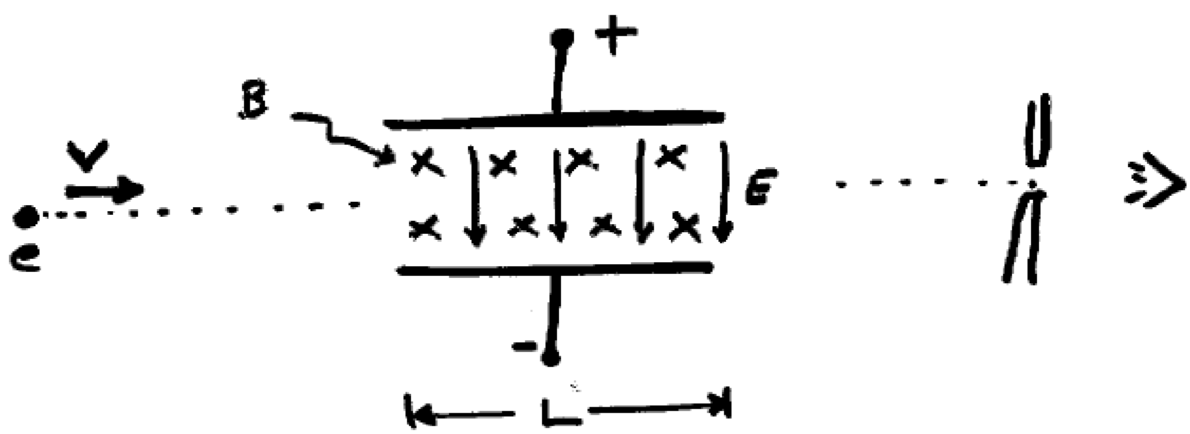
- Both an electric field $\vec{E}$ and a magnetic field $\vec{B}$ can produce a force on a charged particle.
- When the two fields are perpendicular to each other, they are said to be *crossed fields.*
- Here we shall examine what happens to charged particles - namely, electrons - as they move through crossed fields.
- We use as our example the experiment that led to the discovery of the electron in 1897 by J. J. Thomson at Cambridge University.
- Figure shows a modern, simplified version of Thomson's experimental apparatus - a cathode ray tube (which is like the picture tube in an old type television set).



- Charged particles (electrons) are emitted by a hot filament at the rear of the evacuated tube and are accelerated by an applied potential difference *V*.
- After they pass through a slit in screen C, they form a narrow beam.

- They then pass through a region of crossed and fields, headed toward a fluorescent screen S, where they produce a spot of light (on a television screen the spot is part of the picture).
- The forces on the charged particles in the crossed-fields region can deflect them from the center of the screen.
- By controlling the magnitudes and directions of the fields, Thomson could thus control where the spot of light appeared on the screen.
- Recall that the force on a negatively charged particle due to an electric field is directed opposite the field.
- Thus, for the arrangement of in above Fig., electrons are forced up the page by electric field and down the page by magnetic field; that is, the forces are in opposition. Thomson's procedure was equivalent to the following series of steps.

1. Set E = 0 and B = 0 and note the position of the spot on screen S due to the un-deflected beam.
2. Turn on and measure the resulting beam deflection.
3. Maintaining , now turn on and adjust its value until the beam returns to the un-deflected position.(With the forces in opposition, they can be made to cancel.)



- The strength of the magnetic field is increased and adjusted until the incident particle does not experience any vertical deflection.

No deflection implies: $eE = evB$

$$v = \frac{E}{B} \quad (1)$$

- When the magnetic field is turned off the particle is deflected vertically by a distance y_d whose relationship with v is

$$v^2 = \left(\frac{e}{m}\right)\frac{E}{2}\frac{L^2}{y_d} \quad (2)$$

From experiments (1) and (2) we obtain

$$\frac{E^2}{B^2} = \left(\frac{e}{m}\right)\frac{E}{2}\frac{L^2}{y_d}$$

$$\Rightarrow \boxed{\frac{e}{m} = \frac{B^2}{E}\frac{L^2}{2y_d}}$$

J. J. Thomson
1897

Crossed Fields: The Hall Effect

As we know that, a beam of electrons in a vacuum can be deflected by a magnetic field.

Can the drifting conduction electrons in a copper wire also be deflected by a magnetic field?

In 1879, Edwin H. Hall, then a 24-year-old graduate student at the Johns Hopkins University, showed that they can.

This **Hall effect** allows us to find out whether the charge carriers in a conductor are positively or negatively charged. Beyond that, we can measure the number of such carriers per unit volume of the conductor.

Figure 28-8a shows a copper strip of width d, carrying a current *i* whose conventional direction is from the top of the figure to the bottom.

The charge carriers are electrons and, as we know, they drift (with drift speed v_d) in the opposite direction, from bottom to top.

At the instant shown in Fig. 28-8a, an external magnetic field $\vec{B}$, pointing into the plane of the figure, has just been turned on.

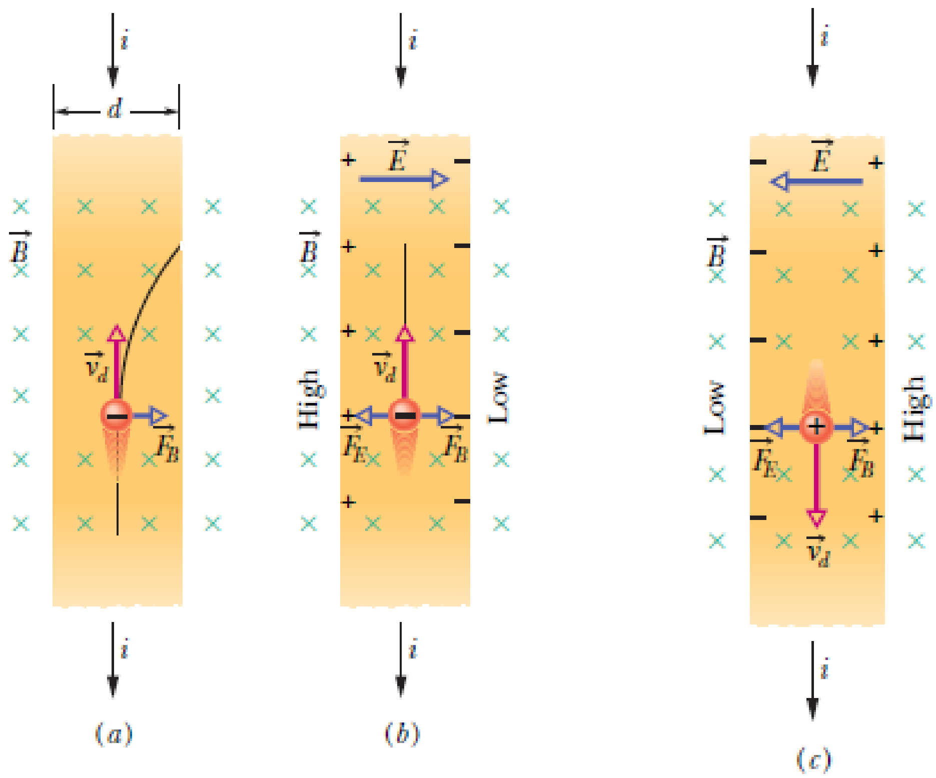
From Eq. 28-2 (F= qvbsinθ) we see that a magnetic deflecting force $\vec{F}$ will act on each drifting electron, pushing it toward the right edge of the strip.

As time goes on, Electrons move to the right, mostly piling up on the right edge of the strip, leaving uncompensated positive charges in fixed positions at the left edge.

The separation of positive charges on the left edge and negative charges on the right edge produces an electric field within the strip, pointing from left to right in Fig. 28-8b.

This field exerts an electric force on each electron, tending to push it to the left.

Thus, this electric force on the electrons, which opposes the magnetic force on them, begins to build up.



Equilibrium quickly develops in which the electric force on each electron has increased enough to match the magnetic force.

When this happens, as Fig. 28-8*b* shows, the force due to $\vec{B}$ and the force due to $\vec{E}$ are in balance.

The drifting electrons then move along the strip toward the top of the page at velocity with no further collection of electrons on the right edge of the strip and thus no further increase in the electric field $\vec{E}$.

Hall potential difference *V* is associated with the electric field across strip width *d*.

$$V = Ed.$$

By connecting a voltmeter across the width, we can measure the potential difference between the two edges of the strip.

Moreover, the voltmeter can tell us which edge is at higher potential.

For the situation of Fig. 28-8*b*, we would find that the left edge is at higher potential, which is consistent with our assumption that the charge carriers are negatively charged.

For a moment, let us make the opposite assumption, that the charge carriers in current *i* are positively charged (Fig. 28-8*c*).

Convince yourself that as these charge carriers move from top to bottom in the strip, they are pushed to the right edge by $\vec{F}$ and thus that the *right* edge is at higher potential.

Because that last statement is contradicted by our voltmeter reading, the charge carriers must be negatively charged.

When the electric and magnetic forces are in balance (Fig. 28-8*b*)

$$eE = ev_d B.$$

The drift speed is

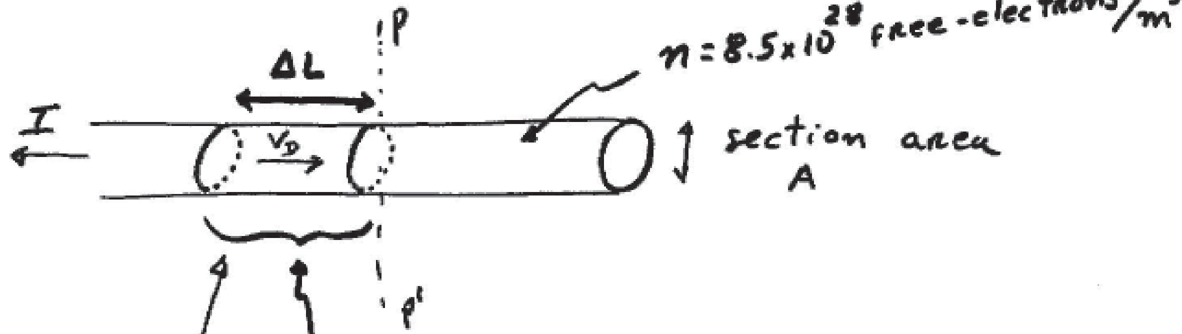
$$v_d = \frac{J}{ne} = \frac{i}{neA}$$

We get

$$n = \frac{Bi}{Vle}$$

Magnetic force on wire that carries current

First, let's make a connection between the current I and the drift speed.



amount of charge in this volume

$$\Delta q = ne \times \text{volume} = ne(\Delta L)A \qquad (1)$$

All this charge will cross the plane PP' in a time Δt

$$\Delta t = \frac{\Delta L}{v_D} \qquad (2)$$

From (1) and (2) we can find I

$$I = \frac{\Delta q}{\Delta t} = n e v_D A$$

I: current; e: electron charge; v_D: drift speed; A: section area of the wire

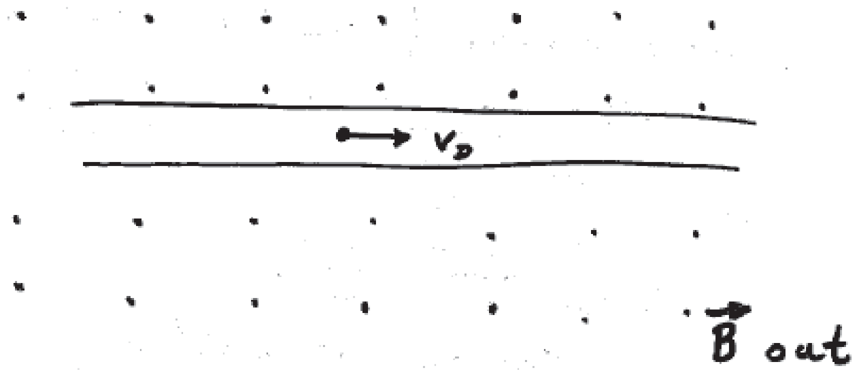
OR

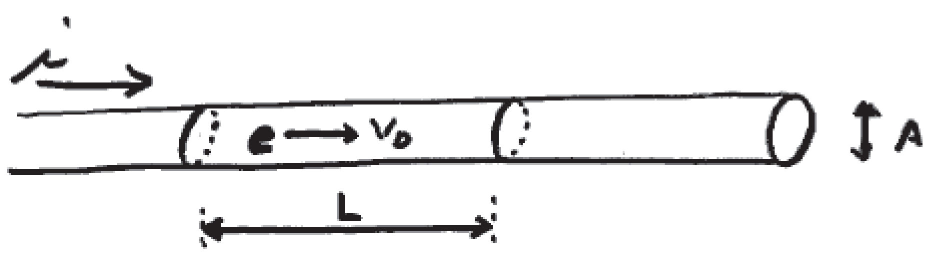
$$j = \frac{I}{A} = n e v_D$$

$e = 1.6 \times 10^{-19}\,C$

n: # of free electrons per m^3

Let's place the wire in a region where it exists a magnetic field





Let's consider just a segment of length "L" of the wire

the force on each charge e is

$e v_D B$

the number of charge in the wire segment is

$$n L A$$

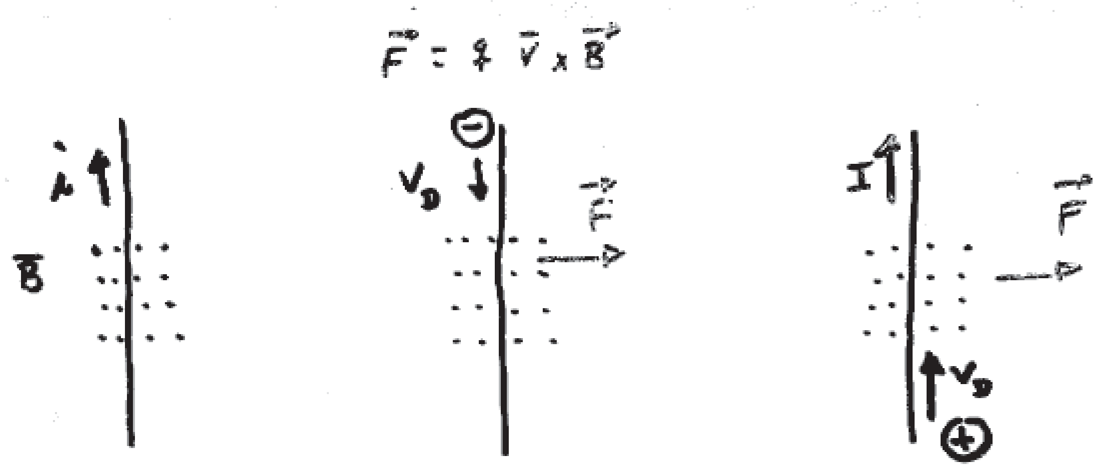
So, the total force is

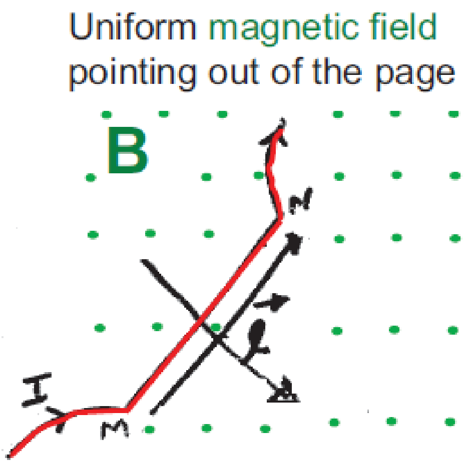
$$F = (e v_D B) n L A$$

Re-arranging the terms, we obtain

$$F = \underbrace{n e v_D A}_{I} \cdot B L$$

$$F = I B L$$





Force on the segment MN

$$\vec{F} = I\,\vec{\ell} \times \vec{B}$$

current

magnetic field (vector)

Force on the small segment $d\vec{\ell}$

$$\Delta\vec{F} = I\,d\vec{\ell} \times \vec{B}$$

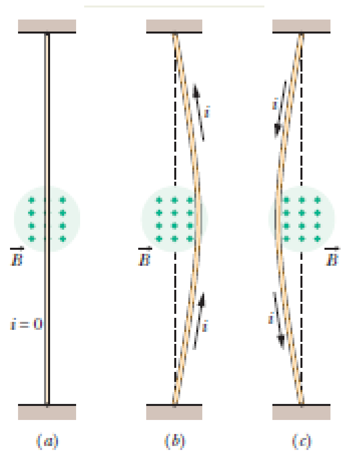
practice: checkpoint #5, p.675
sample problem 29-6 p.675

Magnetic Force on a Current-Carrying Wire

In Fig. 28-14a, a vertical wire, carrying no current and fixed in place at both ends, extends through the gap between the vertical pole faces of a magnet. The magnetic field between the faces is directed outward from the page.

In Fig. 28-14b, a current is sent upward through the wire; the wire deflects to the right.

In Fig. 28-14c, we reverse the direction of the current and the wire deflects to the left.



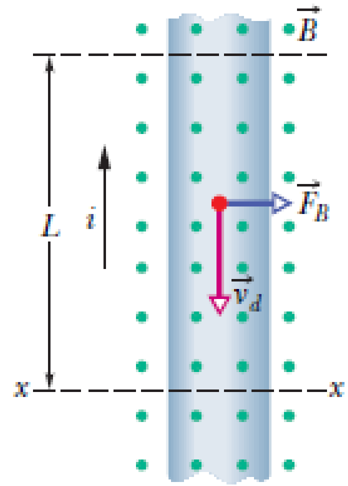


Figure 28-15 shows what happens inside the wire of Fig. 28-14b.

We see one of the conduction electrons, drifting downward with

an assumed drift speed vd.

Equation 28-3, in which we must put θ = 90°, tells us that a force of

Magnitude ev_dB must act on each such electron.

From Eq. 28-2 we see that this force must be directed to the right.

We expect then that the wire as a whole will experience

a force to the right, in agreement with Fig. 28-14b.

We were to reverse either the direction of the magnetic field or the direction of the current, the force on the wire would reverse, being directed now to the left.

Note too that it does not matter whether we consider negative charges drifting downward in the wire (the actual case) or positive charges drifting upward.

The direction of the deflecting force on the wire is the same.

Consider a length L of the wire in Fig. 28-15. All the conduction electrons in this section of wire will drift past plane *xx* in Fig. 28-15 in a time $t = L/v_d$. Thus, in that time a charge given by

$$q = it = i\frac{L}{v_d}$$

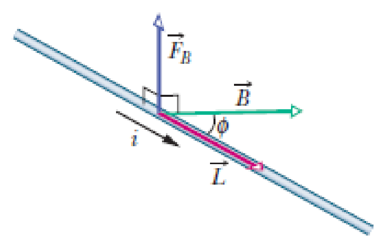
Amount of charges will pass through that plane

$$F_B = qv_dB\sin\phi = \frac{iL}{v_d}v_dB\sin 90^\circ$$

$$F_B = iLB.$$

Note that this equation gives the magnetic force that acts on length L of straight wire carrying a current i and immersed in a Uniform magnetic field that is perpendicular to the wire.

If the magnetic field is not perpendicular to the wire, as in Fig. 28-16, the magnetic force is given by a generalization of Eq.



$$\vec{F}_B = i\vec{L}\times\vec{B}$$

Here $\vec{L}$ is a length vector that has magnitude L and is directed along the wire segment in the direction of the current. The force magnitude F_B is

$$F_B = iLB\sin\phi.$$

ϕ is the angle between the directions of $\vec{L}$ and $\vec{B}$.

The direction of F_B is that of the cross product $\vec{L}\times\vec{B}$ because we take current i to be a positive quantity.

F_B is always perpendicular to the plane defined by vectors $\vec{L}$ and $\vec{B}$

If a wire is not straight or the field is not uniform, we can imagine the wire broken up into small straight segments.

The force on the wire as a whole is then the vector sum of all the forces on the segments that make it up. In the differential limit, we can write

$$d\vec{F}_B = i\, d\vec{L} \times \vec{B},$$

We can find the resultant force on any given arrangement of currents by integrating the above Eq. over that arrangement.

Example

A straight, horizontal length of copper wire has a current I = 28 A through it. What are the magnitude and direction of the minimum magnetic field $\vec{B}$ needed to suspend the wire - that is, to balance the gravitational force on it? The linear density (mass per unit length) of the wire is 46.6 g/m.

Discussion

Because the wire carries a current, a magnetic force can act on the wire if we place it in a magnetic field. To balance the downward gravitational force on the wire, we want F to be directed upward as shown in Fig.

The direction F_B is related to the directions of B and the wire's length vector $\vec{L}$, (F_B = iL × B)

Because $\vec{L}$ is directed horizontally (and the current is taken to be positive)

Right-hand rule for cross products tell us B that must be horizontal and rightward to give the required upward F_B

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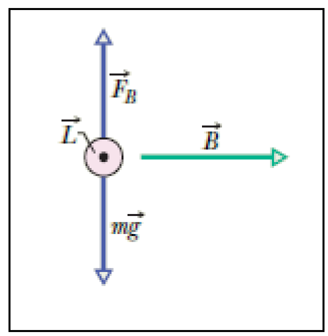


The magnitude of $\vec{F}_B$ is $F_B = iLB \sin \phi$
Because we want $\vec{F}_B$ to balance $\vec{F}_g$, we want

$$iLB \sin \phi = mg,$$

$$\phi = 90°,$$

$$B = \frac{mg}{iL \sin \phi} = \frac{(m/L)g}{i}.$$

$$B = \frac{(46.6 \times 10^{-3}\ \text{kg/m})(9.8\ \text{m/s}^2)}{28\ \text{A}}$$
$$= 1.6 \times 10^{-2}\ \text{T}.$$

This is about 160 times the strength of Earth's magnetic field.

Torque on a Current Loop

Figure shows a simple motor, consisting of a single current-carrying loop immersed in a magnetic field.

The two magnetic forces and produce a torque on the loop, tending to rotate it about its central axis.

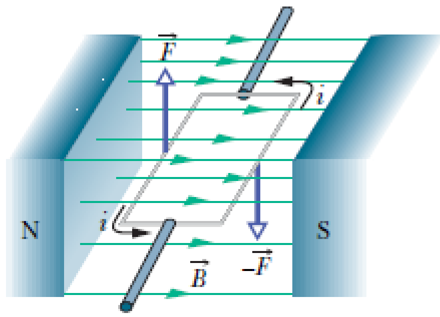
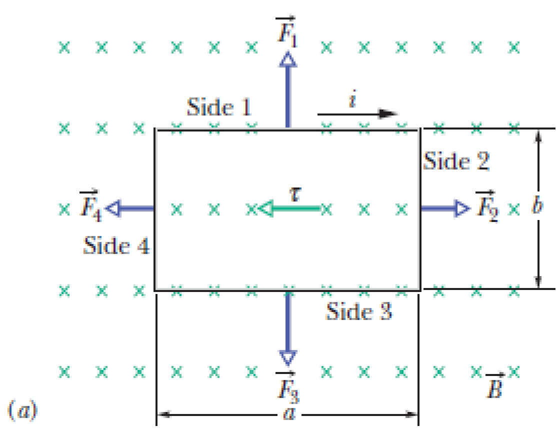


Figure 28-19*a* shows a rectangular loop of sides *a* and *b*, carrying current *i* through uniform magnetic field B.

We place the loop in the field so that its long sides, labeled 1 and 3, are perpendicular to the field direction (which is into the page), but its short sides, labeled 2 and 4, are not.

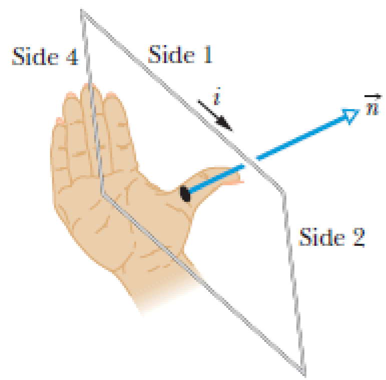
Wires to lead the current into and out of the loop are needed but, for simplicity, are not shown.



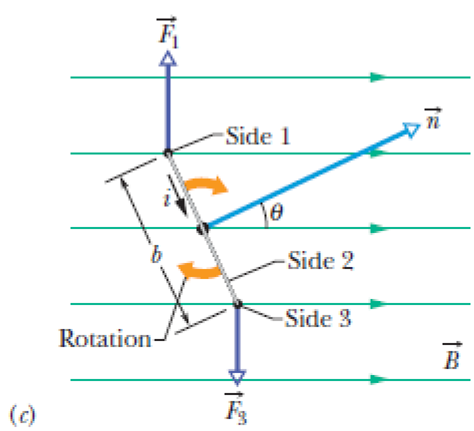
To define the orientation of the loop in the magnetic field, we use a normal vector $\vec{n}$ that is perpendicular to the plane of the loop.

Figure 28-19*b* shows a right-hand rule for finding the direction of $\vec{n}$. Point or curl the fingers of your right hand in the direction of the current at any point on the loop.

Your extended thumb then points in the direction of the normal vector $\vec{n}$.



In Fig. 28-19*c*, the normal vector of the loop is shown at an arbitrary angle u to the direction of the magnetic field .We wish to find the net force and net torque acting on the loop in this orientation.



The net force on the loop is the vector sum of the forces acting on its four sides.

For side 2, the vector in $\vec{L}$ points in the direction of the current and has magnitude *b*. The angle between $\vec{L}$ and $\vec{B}$ for side 2 is 90° - θ. Thus, the magnitude of the force acting on this side is

$$F_2 = ibB \sin(90° - \theta) = ibB \cos \theta.$$

You can show that the force F_4 acting on side 4 has the same magnitude as F_2 but the opposite direction.

Thus, F_2 and F_4 cancel out exactly.

Their net force is zero and, because their common line of action is through the center of the loop, their net torque is also zero.

The situation is different for sides 1 and 3.

For them, L is perpendicular to *B*, so the forces F_1 and F_2 have the common magnitude *iaB*. Because these two forces have opposite directions, they do not tend to move the loop up or down.

However, as Fig. 28-19*c* shows, these two forces do *not* share the same line of action; so they *do* produce a net torque.

The torque tends to rotate the loop so as to align its normal vector with the direction of the magnetic field.

That torque has moment arm (*b*/2) sin θ about the central axis of the loop.

The magnitude ***t*** of the torque due to forces F_1 and F_3 is then (see Fig. 28-19*c*)

$$\tau' = \left(iaB\,\frac{b}{2}\sin\theta\right) + \left(iaB\,\frac{b}{2}\sin\theta\right) = iabB\sin\theta.$$

Suppose we replace the single loop of current with a *coil* of *N* loops, or *turns*.

$$\tau = N\tau' = NiabB\sin\theta = (NiA)B\sin\theta,$$

In which A (= ab) is the area enclosed by the coil. The quantities in parentheses (NiA) are grouped together because they are all properties of the coil: its number of turns, its area, and the current it carries.

For example, for the common circular coil, with radius r, we have

$$\tau = (Ni\pi r^2)B \sin \theta.$$

Magnetic dipole moment:

A current-carrying coil is said to be a ***magnetic dipole.***

Moreover, to account for the torque on the coil due to the magnetic field, we assign a **magnetic dipole moment** $\vec{\mu}$ to the coil.

The direction of $\vec{\mu}$ is that of the normal vector $\vec{n}$ to the plane of the coil and thus is given by the same right hand rule.

$$\mu = NiA$$

In which N is the number of turns in the coil, i is the current through the coil, and A is the area enclosed by each turn of the coil. From this equation, with i in amperes and A in square meters, we see that the unit of $\vec{\mu}$ is the ampere–square meter (A.m^2).

Torque can be rewrite as

$$\tau = (Ni\pi r^2)B \sin \theta.$$

$$\tau = \mu B \sin \theta,$$

in which θ is the angle between the vectors $\vec{\mu}$ and $\vec{B}$.

$$\vec{\tau} = \vec{\mu} \times \vec{B},$$

Torque exerted by an electric field on an electric dipole

$$\vec{\tau} = \vec{p} \times \vec{E}.$$

A magnetic dipole in an external magnetic field has an energy that depends on the dipole's orientation in the field.

$$U(\theta) = -\vec{p} \cdot \vec{E}.$$

In strict analogy, we can write for the magnetic case

$$U(\theta) = -\vec{\mu} \cdot \vec{B}.$$

A magnetic dipole has its lowest energy (= - μB cos 0 = -μB) when its dipole moment $\vec{\mu}$ is lined up with the magnetic field. It has its highest energy (= - μB cos 180° = +μB) when is directed opposite the field.