Coulomb Gauge & Lorentz Gauge:

1. Coulomb Gauge: ($\bar{A}$ expressed in terms of "U")

As in magnetostatics we pick

$$\nabla\cdot\bar{A} = 0 \quad —(1)$$

& Also for Poisson's equation

$$\nabla^2 U = -\frac{\rho}{\epsilon_0} \quad —(2)$$

But one doesn't has to be deceived, though unlike electrostatics "U" by itself doesn't tell us about $\bar{E}$, as we have to know "$\bar{A}$" as well Refer $\bar{E} = -\bar{\nabla}U - \frac{\partial\bar{A}}{\partial t}$ (3)

Since the Solution of Poisson's equation corresponds to $U = \frac{1}{4\pi\epsilon_0}\int_v \frac{\rho}{r}\,dv$ —(4)

There is a strange thing about equation (4). It says that potential everywhere is determined by the distribution of charge right now. If an electron is moved in the lab the potential 'U' on the moon instantaneously records this change. This sounds particularly odd in view of relativity, which allows no messenger to travel faster than the speed

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of light. The point is that, "u" by itself is not a physically measurable quantity. At the moon one can measure $\bar{E}$ & that involves $\vec{A}$ as well. Some how it is built in to the vector potential in the Coulomb guage that where as "u" instantaneously reflects all changes in "ρ", the combination $(-\bar{\nabla}u - \frac{\partial \bar{A}}{\partial t})$ does not. $\bar{E}$ will change only after sufficient time has lapsed. (for the news to arrive.

The advantage of Coulomb gauge is that Scalar potential is particularly simple to calculate, the disadvantage (apart from the non-casual appearance of "u" mentioned above) is that $\bar{A}$ is particularly difficult to calculate. The differential equation for $\bar{A}$ is given as

$$\nabla^2 \bar{A} - \epsilon_0 \mu_0 \frac{\partial^2 \bar{A}}{\partial t^2} = -\mu_0 \bar{J} + \epsilon_0 \mu_0 \bar{\nabla}\left(\frac{\partial u}{\partial t}\right)$$

where $\nabla(\bar{\nabla}\cdot\bar{A}) = 0$.

Hence $\bar{A}$ is expressed in terms of "u".

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The Lorentz Gauge:

($\bar{A}$ & U are treated / gauged on equal footings):

In Lorentz gauge we pick

$$\bar{\nabla}\cdot\bar{A} = -\epsilon_0\mu_0\frac{\partial U}{\partial t} \quad \text{——(1)}$$

This is established to eliminate the Last term at L.H.S in the below equation

$$\nabla^2\bar{A} - \epsilon_0\mu_0\frac{\partial^2\bar{A}}{\partial t} - \nabla(\bar{\nabla}\cdot\bar{A}) + \epsilon_0\mu_0\frac{\partial U}{\partial t} = -\mu_0\bar{J} \text{—(2)}$$

Since $\quad \bar{\nabla}(\bar{\nabla}\cdot\bar{A} + \epsilon_0\mu_0\frac{\partial U}{\partial t}) = 0$

$$\bar{\nabla}\cdot\bar{A} + \epsilon_0\mu_0\frac{\partial U}{\partial t} = 0 \quad \text{ignoring constt.}$$

$$\bar{\nabla}\cdot\bar{A} = -\mu_0\epsilon_0\frac{\partial U}{\partial t} \quad \text{so we have}$$

equat (2) be written as

$$\nabla^2 A - \epsilon_0\mu_0\frac{\partial^2\bar{A}}{\partial t^2} = -\mu_0\bar{J} \text{——(3)}$$

Mean while the equation for potential "U" in Poisson's equation will take the form by putting $\bar{\nabla}\cdot A = -\epsilon_0\mu_0\frac{\partial U}{\partial t}$ as

$$\nabla^2 U - \epsilon_0\mu_0\frac{\partial^2 U}{\partial t^2} = -\frac{\rho}{\epsilon_0} \text{——(4)}$$

Poisson's Eqn Note: $\left[\nabla^2 U + \frac{\partial}{\partial t}(\bar{\nabla}\cdot\bar{A}) = -\frac{\rho}{\epsilon_0}\right]$

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The Lorentz guage :

The Special virtue of Lorentz gauge is that it treats potentials U & $\bar{A}$ on equal footings.

The Same differential operator in four dimension is given

$$\left(\nabla^2 - \epsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} \right) = \Box^2 \quad \text{— (5)}$$

& is called dé Alembrtian.

The democratic treatment of U & $\bar{A}$ is particularly nice in the context of special theory of relativity where dé Alembrtian plays the same what the same role as Laplacian or Poisson operators have in Classical Physics

$$\Box^2 \bar{A} = -\mu_0 \bar{J} \quad \text{— (6)}$$

$$\Box^2 U = -\frac{1}{\epsilon_0} \rho \quad \text{— (7)},$$

Equation (6) & (7) are regarded as 4-dimensional forms of Ampere's Law & Poisson's equation respectively.