Frame of Reference:

To describe a physical event we need at least 3-dimensional coordinate system associated with measurement such a system is called a Frame of reference or a Reference frame.

There are two types of frame of reference:

1. **Inertial frame of Reference or non-accelerated reference frame (RF):**

A Reference Frame which moves with uniform velocity is called inertial-RF.

2. **Non-Inertial frame of Reference or Accelerated Reference frame:**

A Reference frame which moves with variable velocity (increasing/decreasing) with time is known as non-Inertial or Accelerated Reference frame.

Invariance & Covariance:

In order to get an intutive idea of the difference between invariance and covariance, let us consider that we have an aquarium tank filled with water & we define rectangular coordinate (x, y, z) to identify each space point in the tank.

We can express scalar space quantity such as temperature of the water at each space point in the tank by the function $T(x, y, z)$.

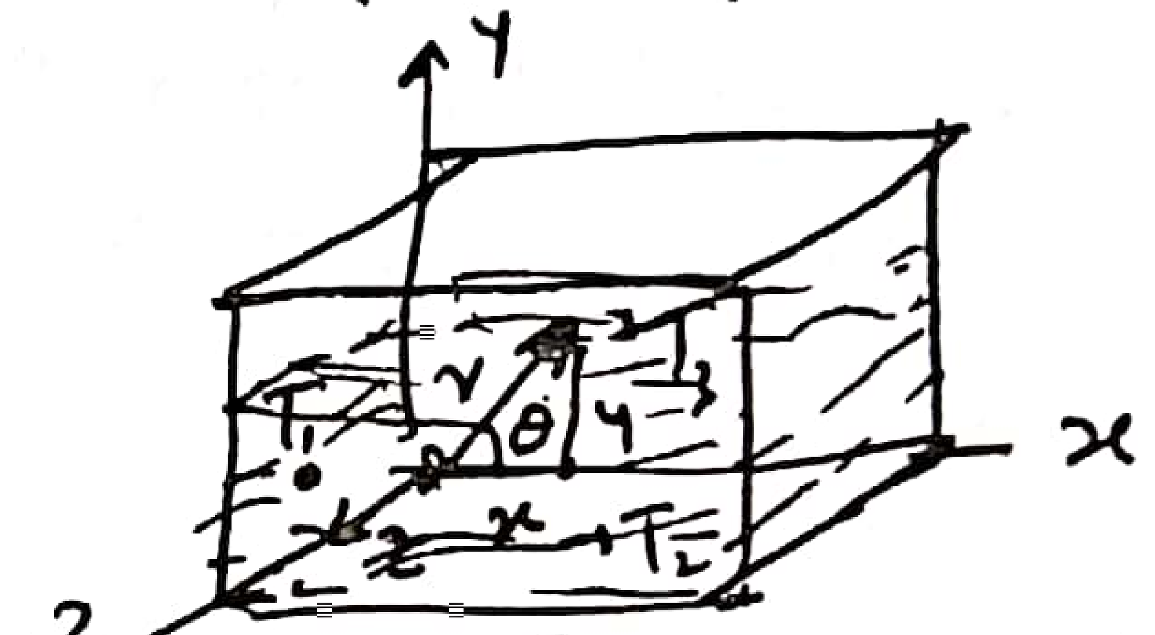

Fig-1 Water tank

This is just a number associated with each point in the tank representing the temperature say in degree Centigrade C° at space points.

Now suppose we change our mind & decide to use spherical polar coordinates instead of Cartesian coordinates.
ie (r, θ, ϕ) Coordinate System — $T(r, \theta, \phi)$

The new coordinates are known as

Contd.

the function of the original coordinates i.e $r(x,y,z)$, $\theta(x,y,z)$ & $\phi(x,y,z)$ Clearly the numerical value of Scalar temperature 'T' at any point in the tank of water is invariant with respect to the changes in coordinates system i.e the values doesn't change with the change of coordinate system hence the quantity 'T' is invariant.

Thus $T(r,\theta,\phi)$ is related to $T(x,y,z)$, where (r,θ,ϕ) are function of (x,y,z).

This means that the numerical value of 'T' at any given point in space is the same (i.e invariant) regardless of the coordinate system we choose.

So we define invariance as,

"If the numerical value of a quantity at a given space point remains the same regardless of the coordinate system we choose, is said to be "invariant" or 'invariance".

Contd:

Covariance:

However suppose we now also measure the gradient $g(x,y,z) = \bar{\nabla} T(x,y,z)$ of the the temperature "T" at each point in space in the water tank. The temperature gradient is vector quantity at each space point (x, y, z) with spatial component

$$\left.\begin{aligned} g_1(x,y,z) &= \frac{\partial}{\partial x} T(x,y,z)\ \hat{x} \\ g_2(x,y,z) &= \frac{\partial}{\partial y} T(x,y,z)\ \hat{y} \\ g_3(x,y,z) &= \frac{\partial}{\partial z} T(x,y,z)\ \hat{z} \end{aligned}\right] - (1)$$

In (r, θ, ϕ) coordinate system

$$\left.\begin{aligned} g_1(r,\theta,\phi) &= \frac{\partial}{\partial r} T(r,\theta,\phi)\ \hat{r} \\ g_2(r,\theta,\phi) &= \frac{1}{r}\frac{\partial}{\partial \theta} T(r,\theta,\phi)\ \hat{\theta} \\ g_3(r,\theta,\phi) &= \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi} T(r,\theta,\phi)\ \hat{\phi} \end{aligned}\right] - (2)$$

where equations (1) & (2) are the partial derivatives. Now let us consider that we want to express spherical polar coordinate system interms of Cartesian coordinate system and vice versa.

Contd.

5/5

The unit vectors expressed as

$$\hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\cos\phi\,\hat{y} + \cos\theta\,\hat{z}$$
$$\hat{\theta} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z} \quad -(3)$$
$$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$$

&

$$\hat{x} = \sin\theta\cos\phi\,\hat{r} + \cos\theta\cos\phi\,\hat{\theta} - \sin\phi\,\hat{\phi}$$
$$\hat{y} = \sin\theta\sin\phi\,\hat{r} + \cos\theta\sin\phi\,\hat{\theta} + \cos\phi\,\hat{\phi} \quad -(4)$$
$$\hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$$

Fortunately it is still possible to express the spatial components $g(r,\theta,\phi)$ in terms of $g(x,y,z)$ but we need to properly take in to account the relation between the spherical & Cartesian coordinate systems

i.e $\quad r = \sqrt{x^2+y^2+z^2} \quad \theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$

$$\phi = \tan^{-1}(y/x)$$

Hence the physical quantities like temperature gradient whose spatial components transform from one coordinate system to the other coordinate system and vice versa, are known as "Covariant".