

Galilean Transformation:

In Galilean Transformation System we consider two Cartesian coordinate systems as shown in Fig-1. Where system - 2 has a constant velocity " v " with respect to system - 1 in the direction of common x -axis.

Considering axes are parallel and are in the same direction. Neither system

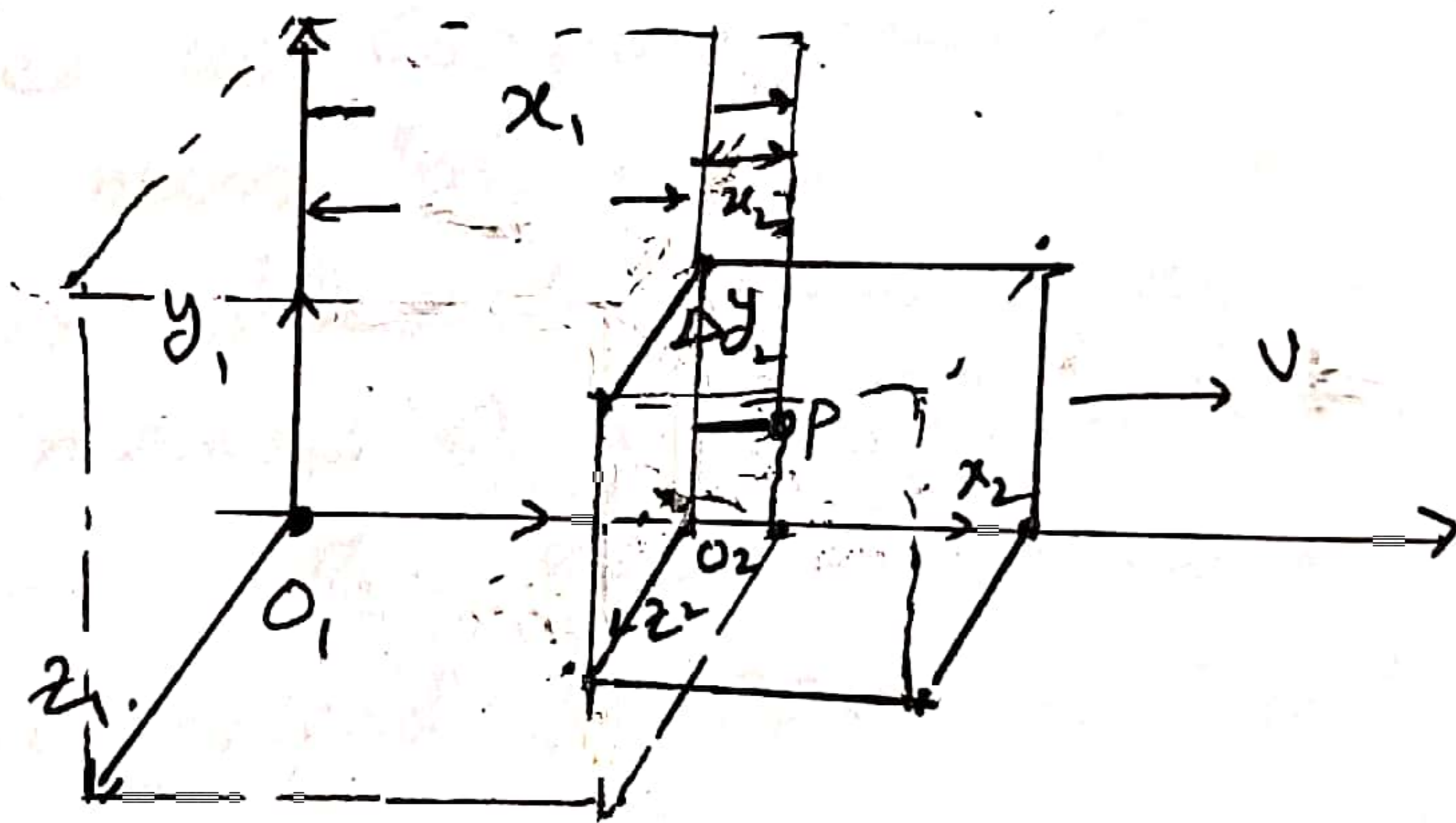


Fig-1: Two Cartesian Coordinate systems one moving with velocity " v " with respect to the other in the x -direction. The two systems overlap when the origins O_1 & O_2 coincide.

is accelerated. The two systems are related

$$\begin{aligned} x_1 &= x_2 + vt & , & & x_2 &= x_1 - vt \\ y_1 &= y_2 & & & y_2 &= y_1 \\ z_1 &= z_2 & & & z_2 &= z_1 \end{aligned}$$

Contd.

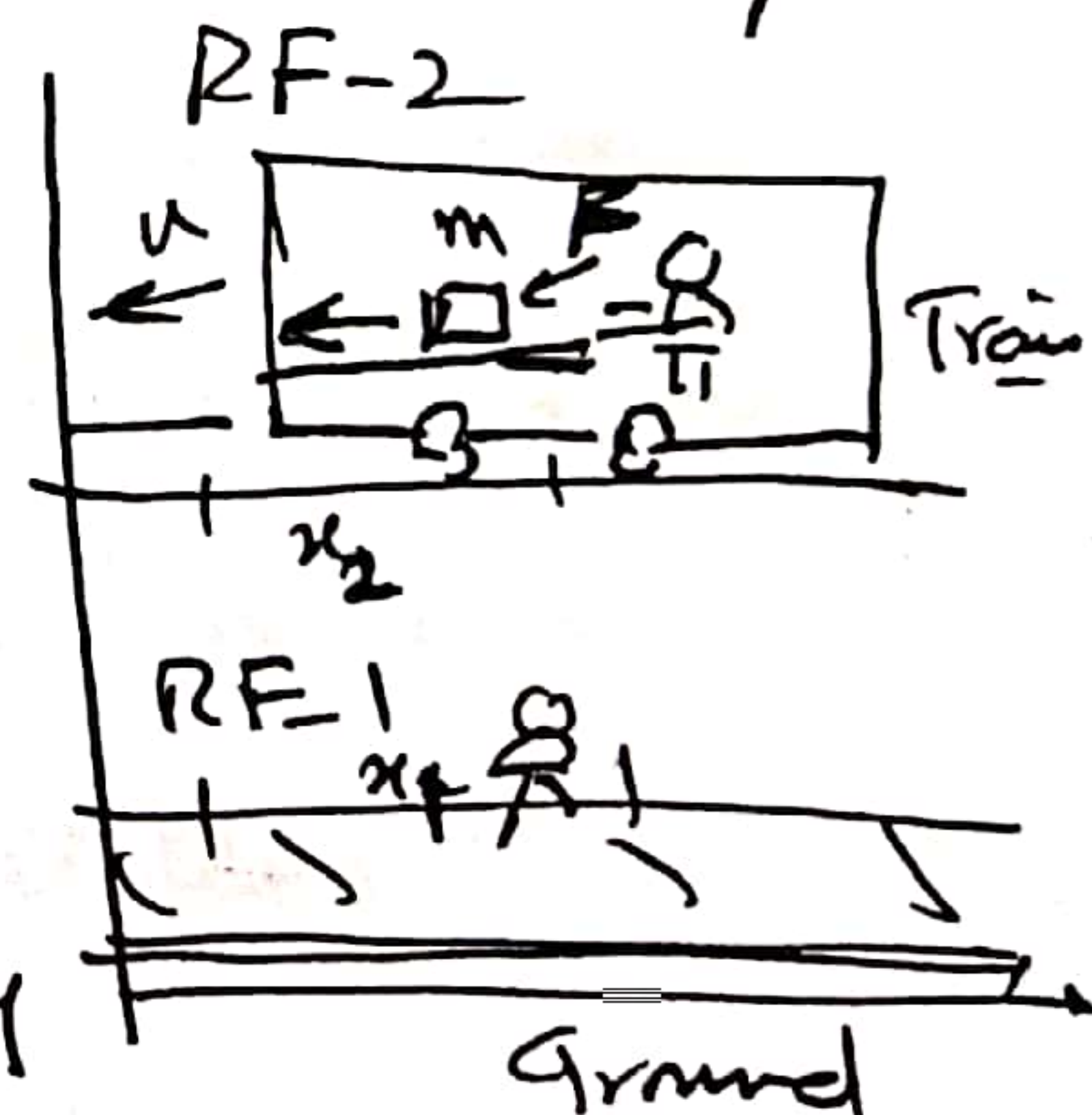
Invariance of $\vec{F} = m\vec{a}$ Law: 2/3

Let us write down the equations for a simple experiment that the passenger performs, first when the train is stopped and then when the train is moving with a constant velocity/speed.

We shall see that equation of motion in RF-1 are of the same as those of in RF-2.

Let us consider that a passenger is given mass " m " on which he exerts a known force F in the direction of the track by means of a calibrated spring in the RF-2. The RF-1 is fixed with respect to the ground & RF-2 is fixed with respect to the train.

We assume that the force F applied a $F = ma$ as observed by RF-1 observer we deduce the corresponding law in RF-2 when the train is stopped we can assume that two frames overlap at $x_1 = x_2$



Contd.,

$$F_1 = ma_1, \quad F_2 = ma_2$$

$$F = m \frac{d^2 x_1}{dt^2} = m \frac{d^2 x_2}{dt^2} \quad \text{--- (1)}$$

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$F = ma$ applies in both systems
When the train is moving with uniform
velocity ' u ' then

$$x_1 = x_2 + ut \quad \text{--- (2)}$$

$$F = m \frac{d^2 x_1}{dt^2} = m \frac{d^2 (x_2 + ut)}{dt^2} \quad \text{--- (3)}$$

$$F = \frac{d^2 x_1}{dt^2} = m \frac{d^2 x_2}{dt^2} \quad \text{--- (4)}$$

$$u = \text{constant} \quad \therefore \frac{d^2 (ut)}{dt^2} = 0$$

Hence $F_2 = F_1$ & the law $F = ma$
applies in both systems.

We therefore say that the law
 $F = ma$ of classical mechanics applies
in both frame of references or that
it is invariant under Galilean Transformations.

$$\text{Similarly } F = m \frac{d^2 y_1}{dt^2} \quad \text{--- (5)}$$

$$F = m \frac{d^2 y_2}{dt^2} \quad \text{--- (6)}$$

$$\text{As } y_1 = y_2$$