Lorentz Transformations for Space and time Coordinates:

Let us Consider two Cartesian Coordinate System RF-1 & RF2 with observers O_1 & O_2 origins. Let RF-2 is moving with uniform velocity 'v'. The event taking place in RF-2 with a Speed of light "c".

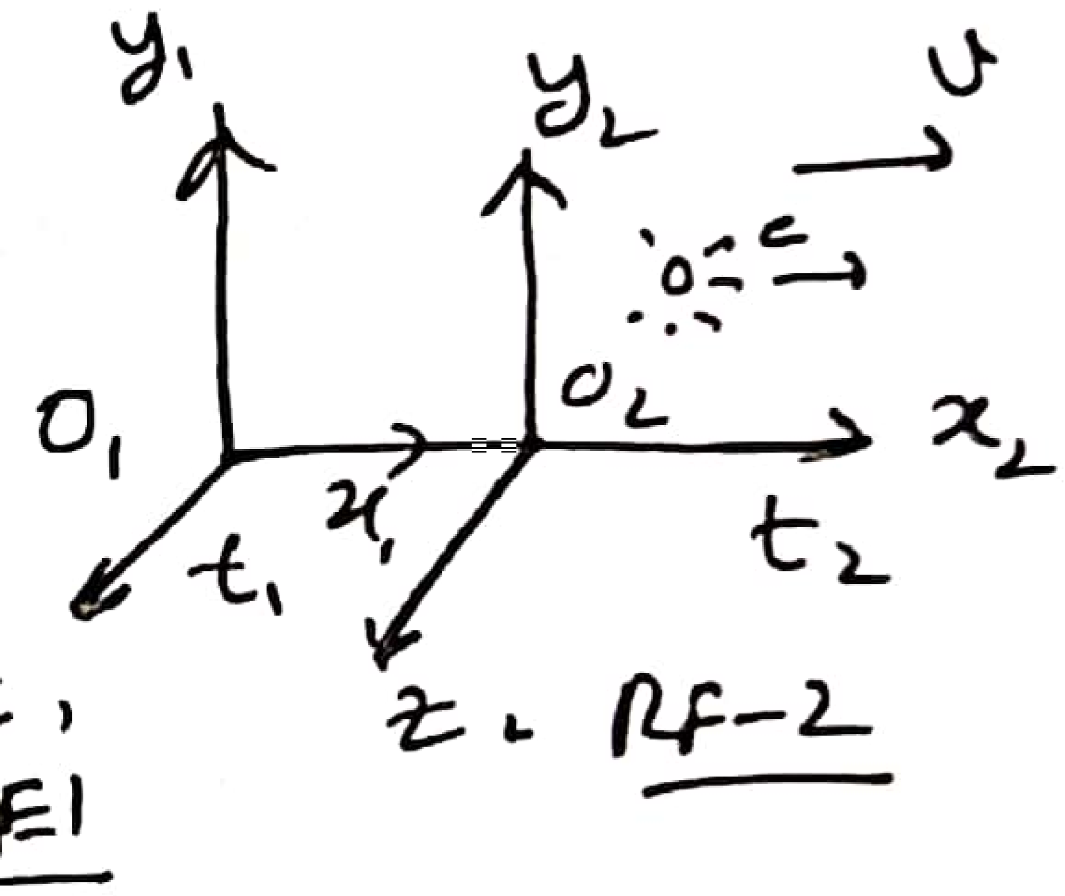

Fig-1

The transformation equations for RF1 & RF-2 are (x_1, y_1, z_1, t_1) & (x_2, y_2, z_2, t_2) respectively.

$$x_1 = \gamma(x_2 + v t_2) \quad \text{---} \quad (1)$$

$$x_2 = \gamma(x_1 - v t_1) \quad \text{---} \quad (2)$$

Where γ is called the relativistic factor. Since the speed of light is independent of source hence "γ" is same for both reference frames RF-1 & RF-2. $\gamma_1 = \gamma_2 = \gamma$

Let
$$x_1 = c t_1 \quad \text{---} \quad (3)$$
$$x_2 = c t_2 \quad \text{---} \quad (4)$$

putting in eq (1) (2) we have

Contd.

$$ct_1 = \gamma\,[ct_2 - vt_2] \quad -(6)$$
$$ct_2 = \gamma\,[ct_1 - vt_1] \quad -(7)$$

From eq (6), we have

$$t_2 = \frac{1}{\gamma}\,\frac{ct_1}{c+v} = \frac{1}{\gamma}\left(\frac{t_1}{1+\frac{v}{c}}\right) \quad -(8)$$

putting t_2 in equation (7), we get

$$c \times \frac{1}{\gamma}\left[\frac{t_1}{1+\frac{v}{c}}\right] = \gamma t_1\,[c - v]$$

Simplifying the above equation

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \quad -(9)$$

Now to find t_2 let us recall equation (1) & (2)

$$x_1 = \gamma\,(x_2 + v t_2) \quad -(1)$$
$$x_2 = \gamma\,(x_1 - v t_1) \quad -(2)$$

putting x_2 from eq (2) in eq (1),

$$x_1 = \gamma\,[\gamma(x_1 - vt_1) + vt_2]$$

Simplifying the above term for t_2 we have

$$t_2 = \frac{x_1}{\gamma v}(1-\gamma^2) + \gamma t_1 \quad -(10)$$

$$\gamma^2 = \frac{1}{1-\left(\frac{v}{c}\right)^2}$$

Hence

$$1-\gamma^2 = 1 - \frac{1}{1-\left(\frac{v}{c}\right)^2}$$

$$= \frac{-\left(\frac{v}{c}\right)^2}{1-\left(\frac{v}{c}\right)^2}$$

$$\text{Simi}\;(1-\gamma^2) = \gamma^2\left(1-\left(\frac{v}{c}\right)^2\right) \quad -$$

Contd

Putting the value of $1-\gamma^2$ in equation (10) & simplifying

$$t_2 = \frac{x_1}{\cancel{\gamma v}}\left[-\gamma^2\left(\frac{v^2}{c^2}\right)\right] + \gamma t_1$$

$$t_2 = \gamma t_1 - x_1 (v/c^2)\gamma$$

$$t_2 = \gamma\left[t_1 - x_1 (v/c^2)\right]$$

$$t_2 = \frac{t_1 - x_1 (v/c^2)}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \qquad (11)$$

Hence we have Lorentz Transformation of Space & time coordinates as:

RF-1	RF-2
$x_1 = \gamma(x_2 + vt_2)$	$x_2 = \gamma(x_1 - vt_1)$
$y_1 = y_2$	$y_2 = y_1$
$z_1 = z_2$	$z_2 = z_1$
$t_1 = \dfrac{t_2 + x_2\left(\frac{v}{c^2}\right)}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$	$t_2 = \dfrac{t_1 - x_1\left(\frac{v}{c^2}\right)}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$