Lorentz Transformations

Length Contraction & Time Dilation:

The Lorentz Transformations forms the basis of Special theory of relativity

The following are the few features of Lorentz Transformations:

(a) – The Lorentz Transformations reduces to the Galilean Trans; if we set the velocity of the object "v" very small as compare to the speed of light $\frac{v}{c} \approx 0$
$v \ll c$

(b) The relative velocity "v" of the two systems can not be greater than "c" otherwise $(x, y, z), t$ become imaginary in one system or the other.

(c) There are really four independent equations. The relation between the quantities in one frame and the corresponding quantities in the other frame can always.

Contd:

be expressed by either one of two equations which are equivalent if the R.H.S can be deduced from the L.H.S

RF-1

$$x_1 = \gamma[x_2 + ut_2]$$
$$y_1 = y_2$$
$$z_1 = z_2$$
$$t_1 = \gamma[t_2 + (\tfrac{v}{c^2})x_2]$$

(RF-2)

$$x_2 = \gamma[x_1 - vt_1]$$
$$y_2 = y_1$$
$$z_2 = z_1$$
$$t_2 = \gamma[t_1 - (\tfrac{v}{c^2})x_1]$$

(d) the Coordinate System in RF-1 is identical to the coordinate system in RF-2 except that subscripts 1 & 2 are interchanged and that '+v' is substituted for '−v'.

<u>General rule</u>: This is termed as a general rule that "If a quantity in one Reference Frame is known in terms of quantities in other frame, the inverse (opposite) relation is obtained by interchanging the subscripts (1) & (2) and changing the sign of "v".

Contd,

Transformation of length
Length Contraction :

Let us consider two frame of references S & S' when $S(x, y, z)$ is a RF-1 stationary while $S'(x', y', z')$ RF-2 is moving with uniform velocity v along the $+x$ direction.

Let a meter rod AB is placed in S' with its proper length "l_0". The same meter rod is observed by the observer in RF-1 i.e. S reference frame as AB but with length "l"

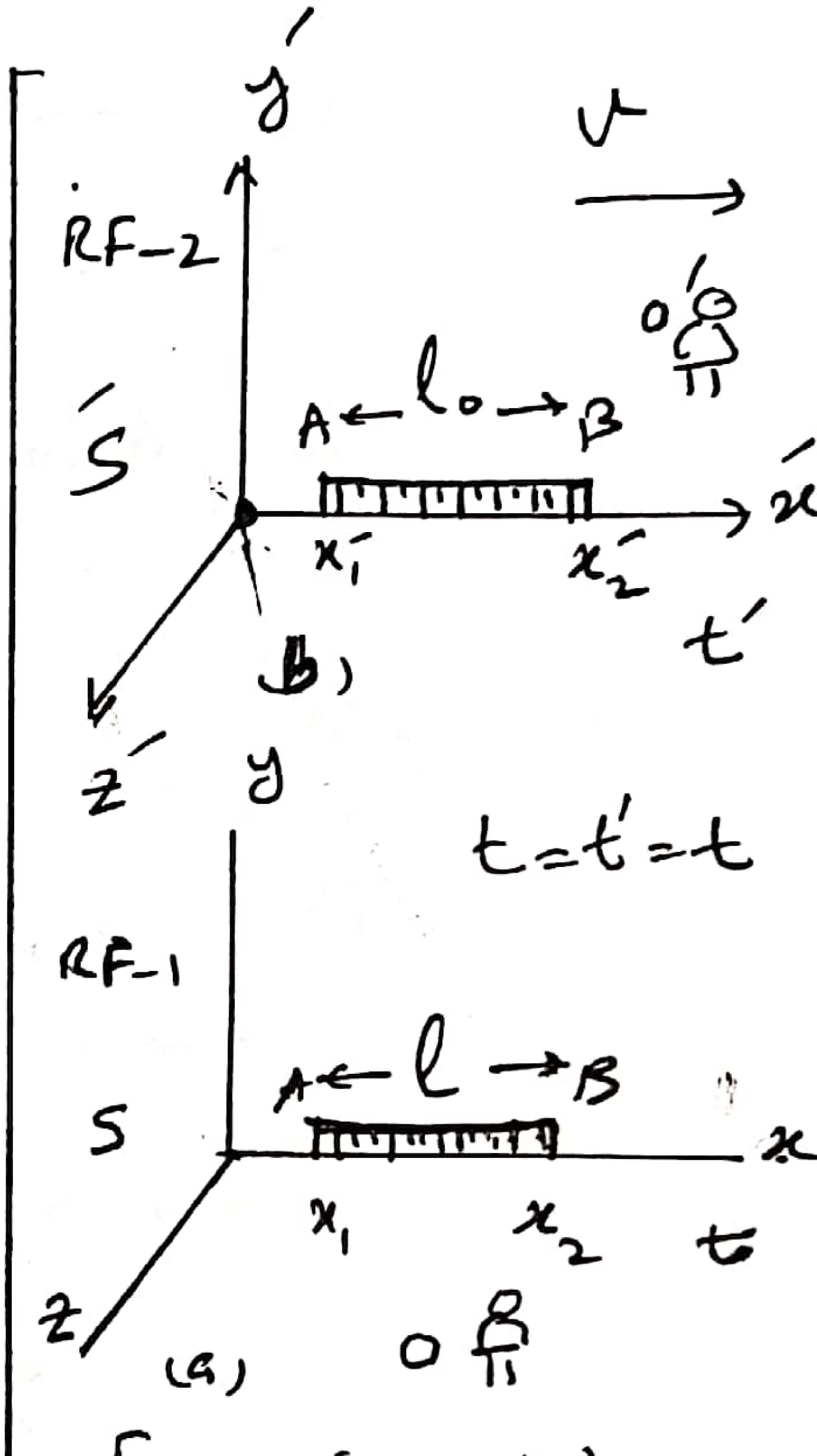


Fig-1 (a & b)

Let the position of the points A & B in S' are given as $A(x_1', y_1', z_1', t)$ & $B(x_2', y_2', z_2', t)$, $t' = t = t$

i.e. the time is same when event occurred.

The change in position is along x-axis only while along other coordinates we have no change i.e. $y_1 = y_1'$, $z_1 = z_1'$

$y_2 = y_2'$ & $z_2 = z_2'$

Contd;

Now let us consider that the length of the rod observed by the observer O' in the S' RF is given as $l_0 = x_2' - x_1'$ —— (1)

& by O in S $l = x_2 - x_1$ — (2)

From the Lorentz Transformation of Space Coordinates we have

$$x_1' = \frac{x_1 - vt}{\sqrt{1-v^2/c^2}} \text{ — (3)}, \quad x_2' = \frac{x_2 - vt}{\sqrt{1-v^2/c^2}} \text{ — (4)}$$

Subtracting (3) from (4) we have

$$x_2' - x_1' = \frac{x_2 - \cancel{vt} - x_1 + \cancel{vt}}{\sqrt{1-v^2/c^2}}$$

$$(x_2' - x_1') = \frac{(x_2 - x_1)}{\sqrt{1-v^2/c^2}} \text{ — (5)}$$ putting (1) & (2) in (5),

$$l_0 = \frac{l}{\sqrt{1-v^2/c^2}} \quad \text{or} \quad l = l_0\sqrt{1-v^2/c^2}$$

$$l = \frac{l_0}{\gamma} \text{ — (6)}$$ Where $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$

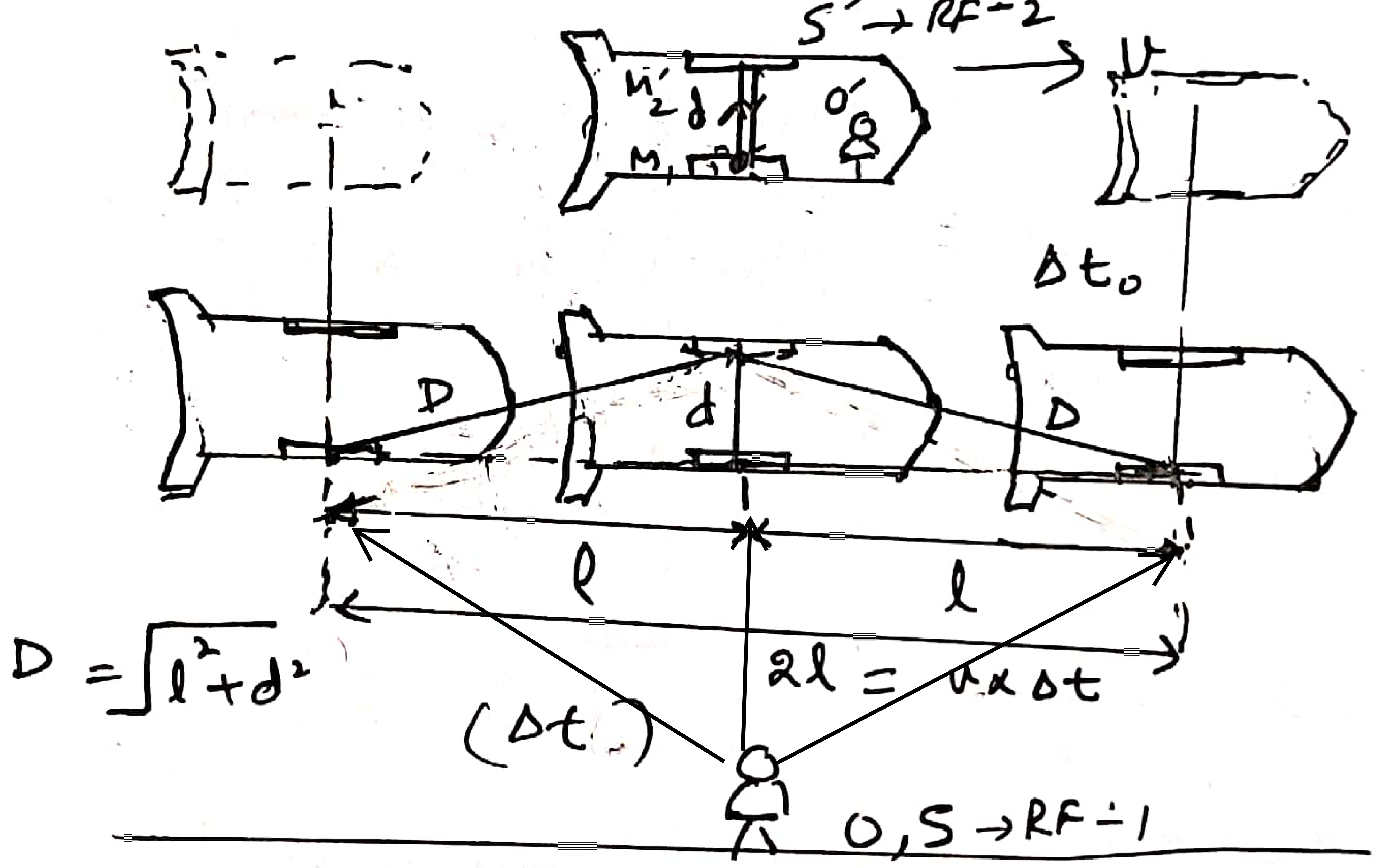
$$\left(\sqrt{1-\frac{v^2}{c^2}} = \frac{1}{\gamma}\right)$$

Hence the length of meter rod observed by the Observer in "S" RF will be seen as contracted. Hence the Length Contraction.

Transformation of time:

Time dilation :

Let us consider a rocket with Reference frame S' having two mirrors $(M_1 M_2)$ mounted at the floor & at the roof. A light pulse originates from M_1 & is reflected back from M_2 travelling distance "d" twice as seen by observer O'



While an observer O is watching this event taking place in a reference from RF-1 or S. Let both observer O & O' have synchronised clocks ... recording times Δt & Δt_0 respectively.

Let RF-2 is moving with a uniform velocity v & observer O' observes during the Δt_0 time that the light travelled "$2d$" distance between M_1 & M_2 mirrors.

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the velocity observed by the O'
observe $\Delta t_0 = \frac{2d}{c}$ —— (1)

While the observer "O" in RF-1
Observer that in $\left(\Delta t = \frac{2l}{v}\right)$ the light travells longer distance given as

$$c = \frac{\text{total Distance}}{\text{total time}} = \frac{2D}{\Delta t} \quad \text{—— (2)}$$

Since $D = \sqrt{l^2+d^2}$ —— (3)

$$c = \frac{2\sqrt{l^2+d^2}}{\Delta t}$$ Also $l = \left(\frac{v\Delta t}{2}\right)$

$$c = \frac{2\sqrt{\left(\frac{v\Delta t}{2}\right)^2 - d^2}}{\Delta t}$$ Also $d = \frac{\Delta t_0 c}{2}$ from eq (1)

$$c = 2\sqrt{\frac{v^2\Delta t^2}{4} - \frac{\Delta t_0^2 c^2}{4}} \quad \text{—— (4)}$$

Squaring the both sides of equation (4), we have

$$c^2 = 4\left(\frac{v^2\Delta t^2 - \Delta t_0^2 c^2}{4}\right)$$

$$c^2 = (v^2\Delta t^2 - \Delta t_0^2 c^2)$$ Dividing both side by c^2

$$1 = \frac{v^2\Delta t^2}{c^2} - \Delta t_0^2$$ Rearranging

$$\left(\frac{\Delta t}{\Delta t_0}\right) = \frac{1}{\sqrt{1 - v^2/c^2}}$$

or $\Delta t = \frac{\Delta t_0}{\sqrt{1-v^2/c^2}} = \gamma \Delta t_0$ —— (5)

Time Dilation takes place.