

Relativistic Doppler effect:

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Let us consider a source 'S' emitting light having the light speed 'c'. Let the source 'S' moves with a uniform velocity 'u' towards an observer 'O' in reference frame-2. Let T' be the time & ν' be the frequency observed by the 'O' observer.

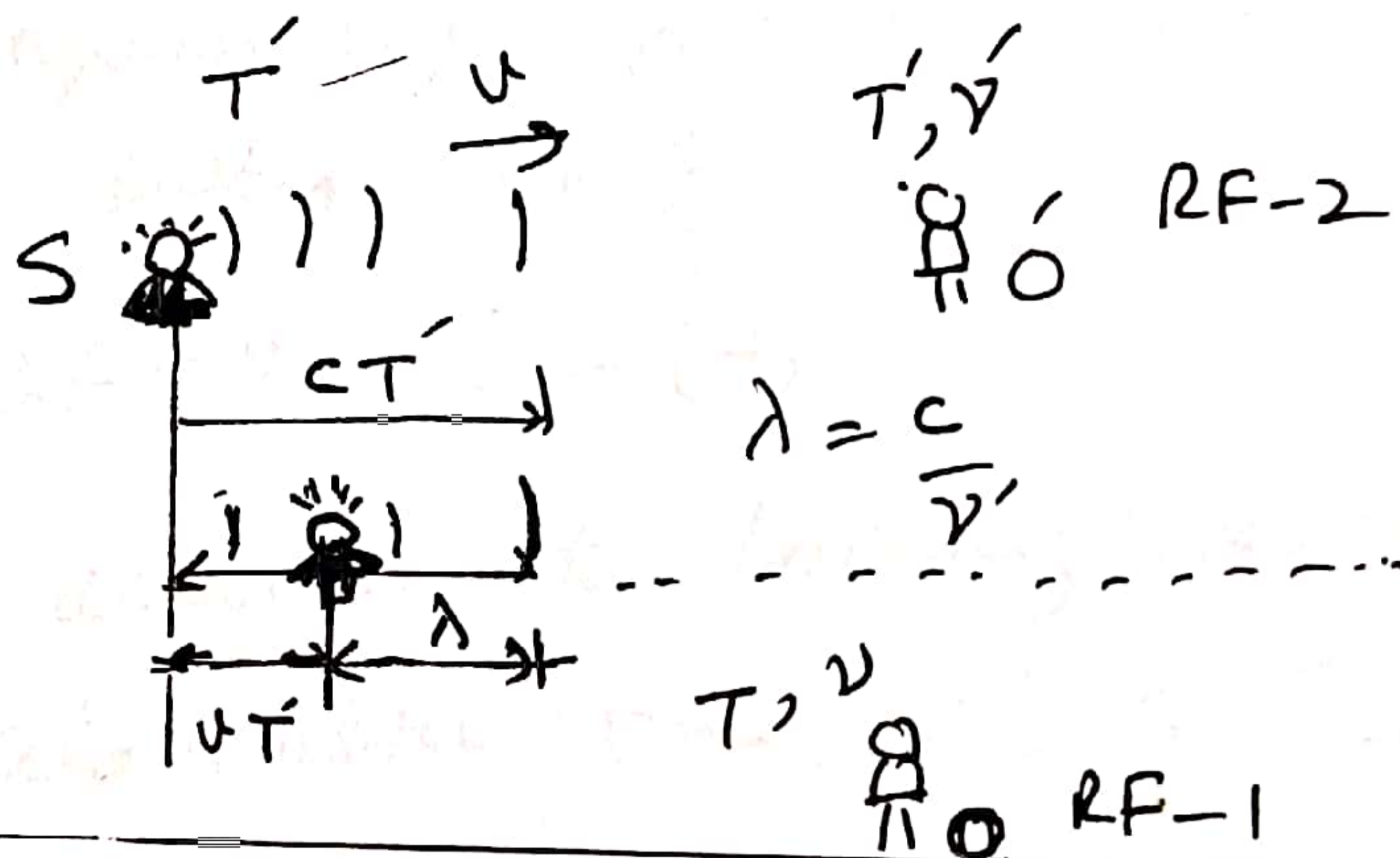


Fig-1

Let an observer "O" in RF-1 which is stationary is observing this whole event & observe T time & ν frequency of the source being observed by O.

Since the speed of light is independent of source & is a constant in all inertial frame of reference.

The wave length " λ " observed
is given as $\lambda = cT' - uT \quad (1)$

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∴ λ is the distance travelled
by the light " cT' " minus " uT ".

$$\text{Since } T' = \frac{1}{\nu'} \quad \& \quad T = \frac{1}{\nu}$$

$$\& \quad \lambda = \frac{c}{\nu'} \quad (2)$$

$$\text{from eqn (1)} \quad \lambda = (c - u)T' \quad (3)$$

$$\text{putting } \lambda = \frac{c}{\nu'}$$

$$\frac{c}{\nu'} = (c - u)T' \quad (4)$$

But according to Lorentz Transformation

$$\text{for relativistic time } T' = \gamma T \quad (5)$$

$$\frac{c}{\nu'} = (c - u)\gamma T \quad \text{Rearranging we get}$$

$$\nu' = \frac{c}{c - u} \times \sqrt{1 - \frac{v^2}{c^2}} \times \nu$$

$$\nu' = \frac{\sqrt{c^2 - v^2} \times \nu}{(c - v)} = \frac{\sqrt{c + v} \times \sqrt{c - v} \times \nu}{\sqrt{c - v} \times \sqrt{c + v}}$$

$$\nu' = \sqrt{\frac{c + v}{c - v}} \nu \quad (6) \quad \text{where } \nu \text{ is the Doppler frequency}$$

When source is moving away towards the observer
When source is moving away $\nu' = \sqrt{\frac{c - v}{c + v}} \nu \quad (7)$