Relativistic Doppler effect:

Let us consider a source 'S' emitting light having the light speed "c". Let the source "S" moves with a uniform velocity "v" towards an observer O′ in reference frame-2. Let T′ be the time & ν′ be the frequency observed by the O′ observer.

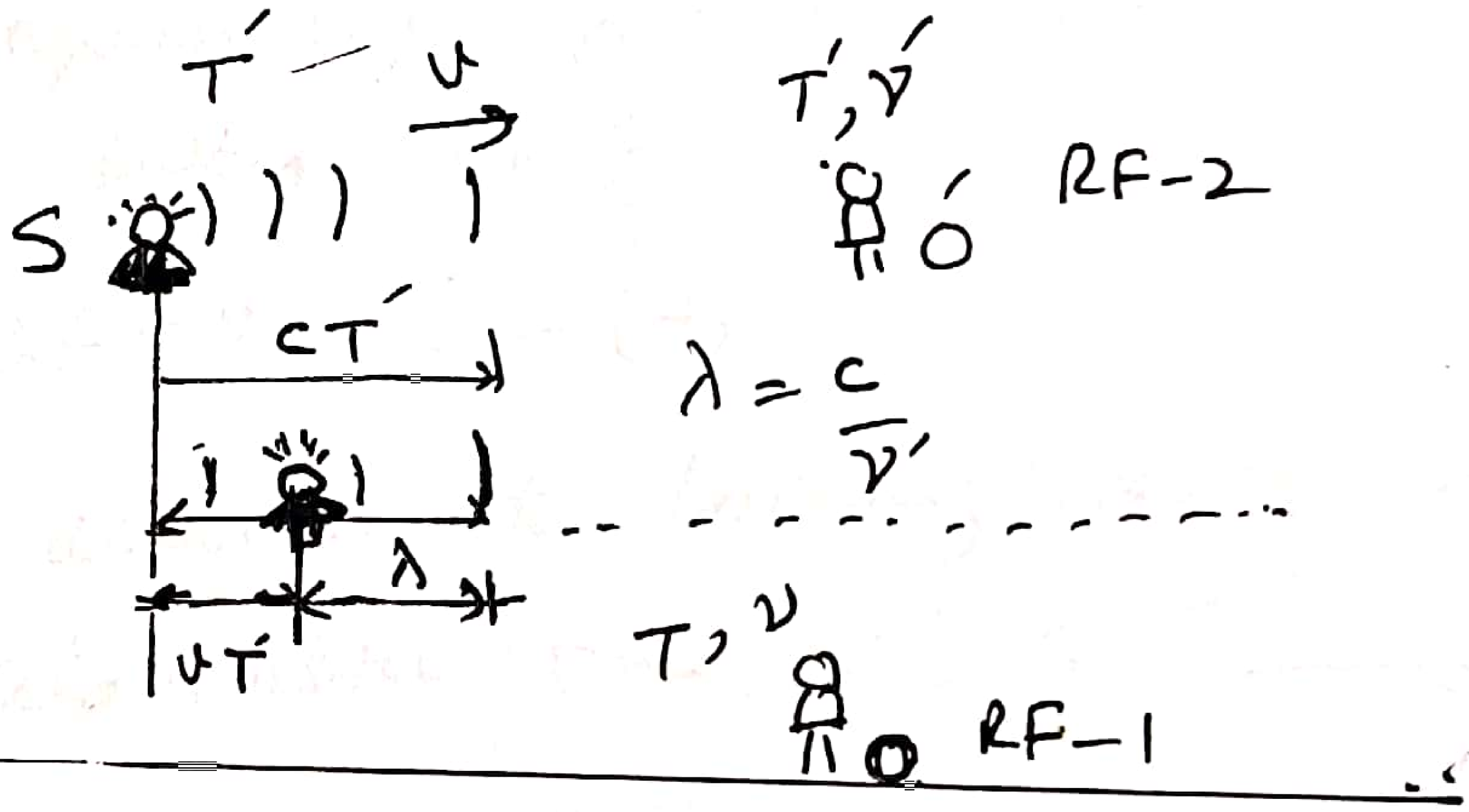

Fig-1

Let an observer "O" in RF-1 which is stationary is observing this whole event & observe T time & ν frequency of the source being observed by O′.

Since the speed of light is independent of source & is a constant in all inertial frame of references.

The wave length "λ" observed is given as $\lambda = cT' - vT'$ —(1)

where λ is the distance travelled by the light "T'" minus "vT'".

Since $T' = \frac{1}{\nu'}$ & $T = \frac{1}{\nu}$

& $\lambda = \frac{c}{\nu'}$ —(2)

from equation (1) $\lambda = (c-v)T'$ —(3)

putting $\lambda = \frac{c}{\nu'}$

$\frac{c}{\nu'} = (c-v)T'$ —(4)

But according to Lorentz Transformation for relativistic time $T' = \gamma T$ —(5)

$\frac{c}{\nu'} = (c-v)\gamma T$ Re-arranging we get

$$\nu' = \frac{c}{c-v} \times \sqrt{1-\left(\frac{v}{c}\right)^2}\,\nu$$

$$\nu' = \frac{\sqrt{c^2-v^2}}{\sqrt{(c-v)^2}} \times \nu = \frac{\sqrt{c+v} \times \cancel{\sqrt{c-v}}}{\sqrt{c-v} \times \cancel{\sqrt{c-v}}} \times \nu$$

$$\nu' = \sqrt{\frac{c+v}{c-v}}\,\nu \quad \text{—(6)}$$

where ν' is the Doppler frequency when source is moving towards the observer

when source is moving away $\nu' = \sqrt{\frac{c-v}{c+v}}\,\nu$ —(7)