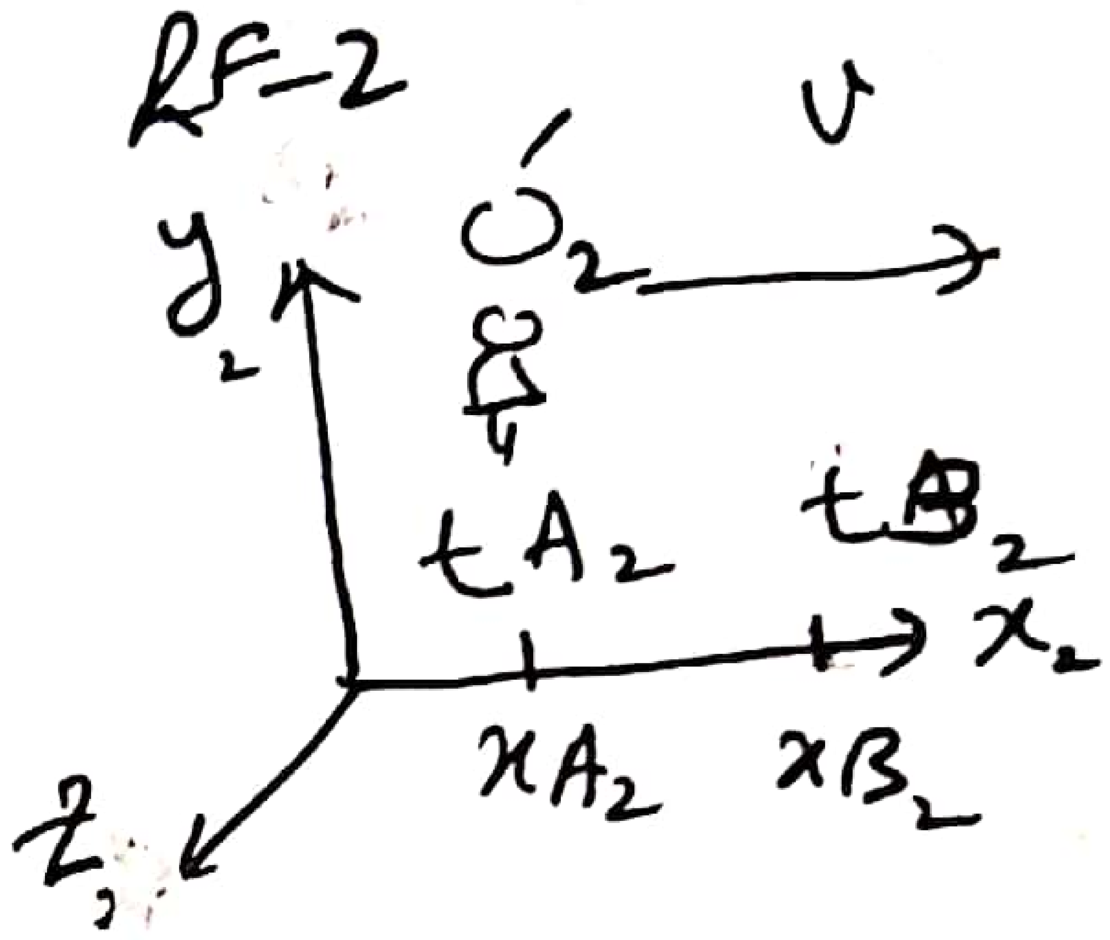
Simultaneity:

Imagine that observer O_1 sees two events A & B that to him occur at the same x-coordinates $x_{A_1} = x_{B_1}$ at the same time $t_{A_1} = t_{B_1}$



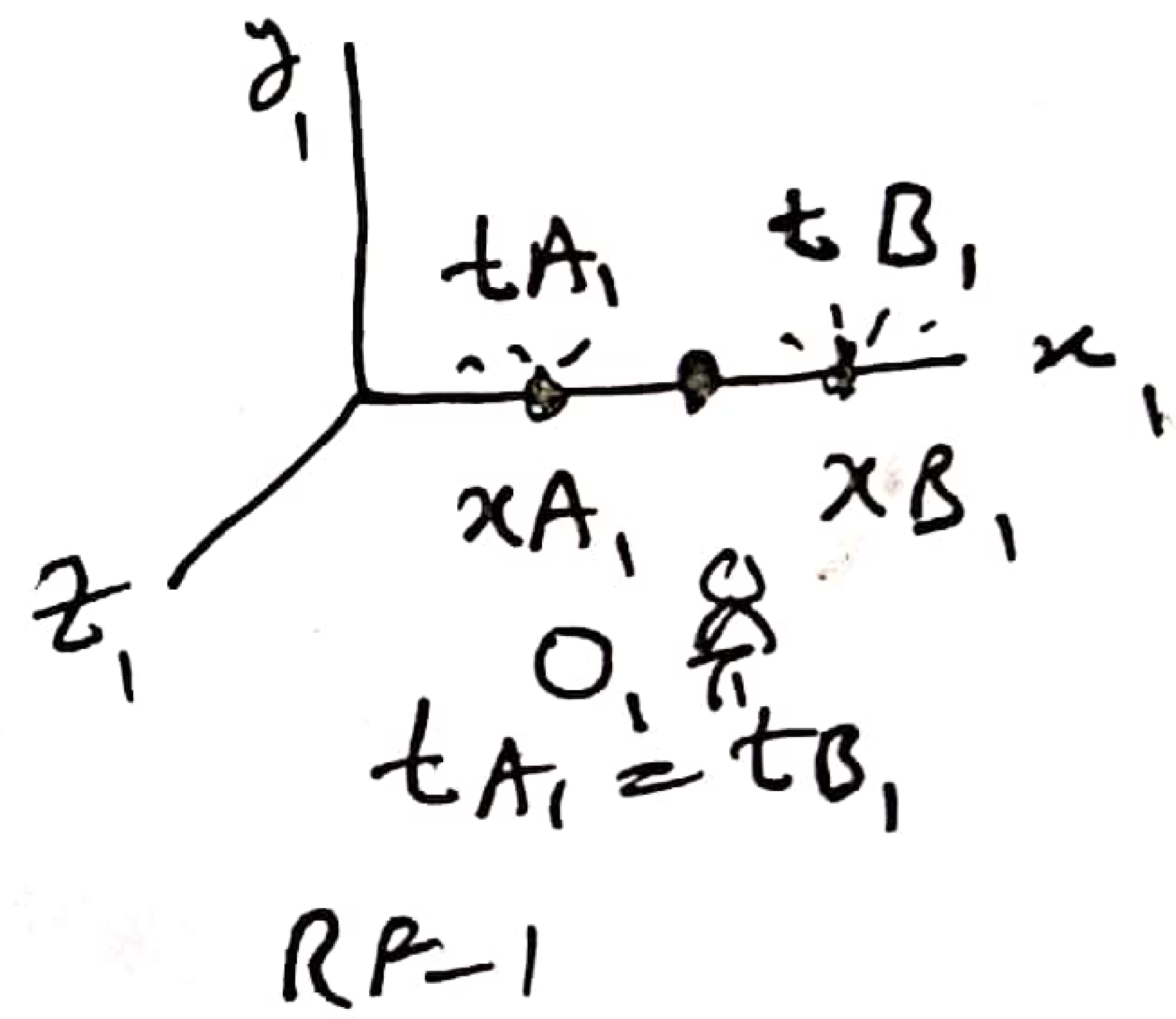


Fig-1

He observes that A & B are simultanous like two fire crackers crack at the same time. Are the events also simultanous for observer O_2 who is moving with uniform velocity / speed v?

From Lorentz Transformation equations for time it will see

$$t_{A_2} = \gamma\left[t_{A_1} - \frac{v}{c^2} x_{A_1}\right] \quad \text{—— (1)}$$

$$t_{B_2} = \gamma\left[t_{B_1} - \frac{v}{c^2} x_{B_1}\right] \quad \text{—— (2)}$$

Contd:

The two events occur at the same value of "x" that are simultaneous for one observer are also simultaneous for another observer provided other observer is also at rest.

But if the other observer is moving with uniform velocity 'v' the two events need not occur at the same place because ys & zs can be different. What if the events donot occur at the same value of x if they are simultaneous for "O_1". Then $t_{A_1} = t_{B_1}$ But since $x_{A_1} \neq x_{A_2}$ They are not simultaneous for the observer "O_2". Ifact the time interval between the two events as seen by O_2 in Reference frame-2 is

$$t_{B_2} - t_{A_2} = \gamma\left[t_{B_1} - \frac{v}{c^2}x_{B_1}\right] - \gamma\left[t_{A_1} - \frac{v}{c^2}x_{A_1}\right]$$

as $t_{B_1} = t_{A_1} = t$

$$= \gamma\left\{\left[\cancel{t_{B_1} - t_{A_1}}\right] + (x_{A_1} - x_{B_1})\frac{v}{c^2}\right]$$

$$(t_{B_2} - t_{A_2}) = \gamma\left[(x_{A_1} - x_{B_1})\frac{v}{c^2}\right] \quad (3)$$

Observer "O_2" will observe that cracker at x_{A_1} will creak first & then cracker at x_{B_1} will crack after that interval.