Transformation of Velocity:

Let us consider that in Reference frame-2 which is moving with uniform velocity v along positive x-axis. An observer O_2 in the RF-2 observes that an object in the RF-2 is moving with velocity V_{2x} also along the positive x-axis. What will be the velocity of the same object according to the observer "O_1" in the reference frame-1 which is at rest. Note that the V_{2x} need not be constant.

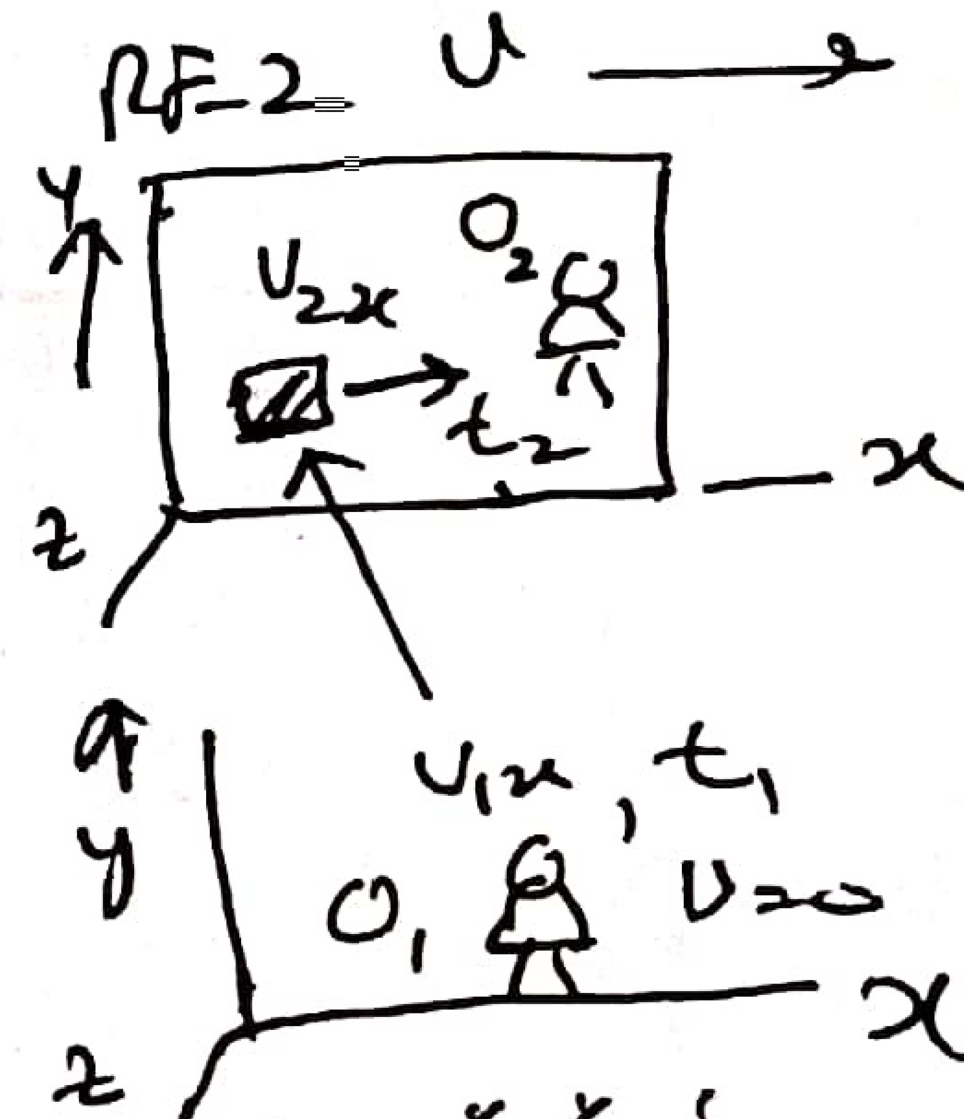

Fig-1

Let the velocity of the object as observed by the observer "O_1" be $V_{1x} = \dfrac{dx_1}{dt_1}$ as shown in Fig-1.

The Lorentz Transformation equation for space coordinates along "x" is given as

$$x_1 = \gamma(x_2 + vt_2) \quad \text{——} \quad (1)$$

Time 4

$$t_1 = \gamma\left(t_2 + \frac{v}{c^2}x_2\right) \quad \text{——} \quad (2)$$

Contd;

For any Change of any change of displacement or time the equations (1) & (2) can be written as

$$dx_1 = \gamma\,(dx_2 + v\,dt_2) \quad —(3)$$

$$dt_1 = \gamma\,\{dt_2 + (v/c^2)\,dx_2\} \quad —(4)$$

Dividing equation (3) by equation (4) we get

$$\frac{dx_1}{dt_1} = V_{1x} = \frac{\gamma\,(dx_2 + v\,dt_2)}{\gamma\,(dt_2 + \frac{v}{c^2}\,dx_2} \quad —(5)$$

Multiplying & dividing the L.H.S of equation (5) by $(1/dt_2)$ we have

$$U_{1x} = \left[\frac{\frac{dx_2}{dt_2} + v}{1 + \frac{v}{c^2}\frac{dx_2}{dt_2}}\right]$$

$$V_{1x} = \left[\frac{U_{2x} + v}{1 + \frac{v}{c^2}U_{2x}}\right] \quad —(6)$$

The velocity V_{1x} is measured at the time "t_1" & V_{2x} is measured at the corresponding time 't_2'. Therefore the velocity in RF-1 is smaller than $(U_{2x}+v)$ by a factor $\left(1+\frac{U_{2x}v}{c^2}\right)$.

Now let us see situation for V_{1y} & V_{1z}.

Since $y_1 = y_2$ ⟵ $y_1 = y_2$

& $V_{1y} = \frac{dy_1}{dt_1}$ & $V_{1y} = \frac{dy_2}{dt_1}$

we can express

$$V_{1y} = \frac{dy_2}{dt_1} \times \frac{dt_2}{dt_2}$$ Multiplying & dividing by (dt_2).

$$V_{1y} = \frac{dy_2}{dt_2} \times \left(\frac{dt_2}{dt_1}\right) \quad \text{——} (7)$$

As $t_1 = \gamma\left[t_2 + \frac{v}{c^2}x_2\right]$ —— (8)

Differentiating equation (8) with respect to "t_2"

$$\frac{dt_1}{dt_2} = \gamma\left[1 + \frac{v}{c^2}\frac{dx_2}{dt_2}\right)$$

$$\frac{dt_1}{dt_2} = \gamma\left[1 + \frac{v}{c^2}U_{2x}\right] \quad \text{—} (9)$$

putting eq (9) i.e $\left(\frac{dt_1}{dt_2}\right)$ value in 7,

inverse of eq (9) $\left(\frac{dt_2}{dt_1}\right)$

Contd:

$$V_{1y} = \frac{V_{2y}}{\gamma\left[1 + V_{2x}\frac{v}{c^2}\right]} \quad \text{——— (10)}$$

Similarly for V_{1z} we have

$$V_{1z} = \frac{V_{2z}}{\gamma\left[1 + V_{2x}\frac{v}{c^2}\right]} \quad \text{——— (11)}$$

Hence the Transformation equations for velocity are :

Observer O_1	Observer O_2
$V_{1x} = \dfrac{V_{2x} + v}{1 + \dfrac{v}{c^2} V_{2x}}$	$V_{2x} = \dfrac{V_{1x} - v}{1 - \dfrac{v}{c^2} V_{1x}}$
$V_{1y} = \dfrac{V_{2y}}{\gamma\left[1 + V_{2x}\dfrac{v}{c^2}\right]}$	$V_{2y} = \dfrac{V_{1y}}{\gamma\left[1 - V_{1x}\dfrac{v}{c^2}\right]}$
$V_{1z} = \dfrac{V_{2z}}{\gamma\left[1 + V_{2x}\dfrac{v}{c^2}\right]}$	$V_{2z} = \dfrac{V_{1z}}{\gamma\left[1 - V_{1x}\dfrac{v}{c^2}\right]}$