Transformation of acceleration:

We can transform acceleration as observed by oberver O_1 in a RF-1 which at rest while the object moving in a RF-2 with velocity V_{2x} and also the RF-2 is moving with uniform velocity "V".

The acceleration o[illegible] by the observer "O_1" in RF-1 that of the object moving in RF-2 is given as

$$a_{1x} = \frac{dU_{1x}}{dt_1} \quad \text{———} (1)$$

or

$$a_{1x} = \frac{dv_{1x}}{dt_2} \times \frac{dt_2}{dt_1} \quad \text{———} (2)$$

Also we have

$$U_{1x} = \frac{U_{2x} + V}{\left(1 + \frac{V}{c^2} U_{2x}\right)} \quad \text{———} (3)$$

Differentiation eq (3) w.r.t "t_2"

$$\frac{dU_{1x}}{dt_2} = \frac{d}{dt_2}\left(\frac{U_{2x} + V}{1 + \frac{V}{c^2} U_{2x}}\right)$$

Contd.

$$\frac{dv_{1x}}{dt_2} = \frac{\left(1+\frac{v}{c^2}v_{2x}\right)\frac{d}{dt_2}\left(v_{2x}+v\right) - \left(v_{2x}+v\right)\frac{d}{dt_2}\left(1+\frac{v}{c^2}v_{2x}\right)}{\left(1+\frac{v}{c^2}v_{2x}\right)^2}$$

$$= \frac{\left(1+\frac{v}{c^2}v_{2x}\right)\left(a_{2x}\right) - \left(v_{2x}+v\right)\left(\frac{v}{c^2}a_{2x}\right)}{\left(1+v/c^2\, v_{2x}\right)^2}$$

$$= \frac{\left(a_{2x} + \cancel{a_{2x}\frac{v}{c^2}v_{2x}} - \cancel{a_{2x}\frac{v}{c^2}v_{2x}} - \left(\frac{v}{c}\right)^2 a_{2x}\right)}{\left(1+\frac{v}{c^2}v_{2x}\right)^2}$$

$$\frac{dv_{1x}}{dt_2} = \frac{a_{2x}\left[1-\left(\frac{v}{c}\right)^2\right]}{\left[1+\left(\frac{v}{c}\right)^2\right]^2} \quad \text{—— (4)}$$

(*)

Also we have (Already done) $\frac{dt_2}{dt_1} = \frac{1}{\gamma\left[1+\frac{v}{c^2}v_{2x}\right]}$ —— (5)

Putting equation (4) & (5) in eq (2) we have

$$a_{1x} = \frac{a_{2x}}{\gamma^3\left[1+\frac{v}{c^2}v_{2x}\right]^3} \quad \text{—— (6)}$$

* Note: in eq (4) we put $\left[1+\left(\frac{v}{c}\right)^2\right] = \left(\frac{1}{\gamma^2}\right)$

Contd

Similary for "a_{1y}" let us take

$$u_{1y} = \frac{u_{2y}}{\gamma\left[1 + v_{2x}\left(\frac{v}{c^2}\right)\right]} \quad \text{—— (7)}$$

$$a_{1y} = \frac{du_{1y}}{dt_1} = \frac{du_{1y}}{dt_2} \times \frac{dt_2}{dt_1} \quad \text{—— (8)}$$

$$\frac{d}{dt_2}(u_{1y}) = \frac{d}{dt_2}\left[\frac{u_{2y}}{\gamma\left[1 + v_{2x}\left(\frac{v}{c^2}\right)\right]}\right] \quad \text{—— (9)}$$

Applying differential on eq 7, we get in the similar way

$$= \frac{\left(1 + v_{2x}\frac{v}{c^2}\right)a_{2y} - v_{2y}\left[a_{2x} v/c^2\right]}{\gamma\left(1 + v_{2x} v/c^2\right)^2}$$

$$= \left(a_{2y} + a_{2y}u_{2x}v/c^2 - a_{2x}v_{2y}v/c^2\right) \Big/ \left(1 + v_{2x}\frac{v}{c^2}\right)^2 \gamma$$

$$= \frac{\cancel{\left(1 + v_{2x}v/c^2\right)}\left[a_{2y} - \left(v_{2y}\,v/\cancel{c^2}\,a_{2x}\right)\cancel{c^2} / (c^2 + v_{2x}v)\right]}{\gamma\left[1 + v_{2x}v/c^2\right]^{\cancel{2}}}$$

$$\frac{du_{1y}}{dt_2} = \frac{\left(a_{2y} - \frac{u_{2y}a_{2x}v}{c^2 + v_{2x}v}\right)}{\gamma\left[1 + v_{2x}\frac{v}{c^2}\right]} \quad \text{—— (10)}$$

Also we have

$$\frac{dt_2}{dt_1} = \frac{1}{\gamma\left[1+\frac{v}{c^2}V_{2x}\right]} \quad \text{--- (11)}$$

putting the values of $\frac{dU_{1y}}{dt_2}$ & $\frac{dt_2}{dt_1}$ from equation (10) & (11) in equation (8) we have

$$a_{1y} = \frac{\left(a_{2y} - \frac{U_{2y}a_{2x}v}{c^2+V_{2x}v}\right) \Big/ \gamma\left[1+V_{2x}\frac{v}{c^2}\right]}{\gamma\left[1+V_{2x}\frac{v}{c^2}\right]}$$

Rearranging & Simplify we get

$$a_{1y} = \left(a_{2y} - \frac{V_{2y}a_{2x}v}{c^2+V_{2x}v}\right) \times \frac{1}{\gamma^2\left[1+V_{2x}\frac{v}{c^2}\right]^2} \quad \text{--- (12)}$$

Similarly we ... solve for a_{1z}

$$a_{1z} = \left(a_{2z} - \frac{U_{2z}a_{2x}v}{c^2+V_{2x}v}\right) \times \frac{1}{\gamma^2\left(1+V_{2x}\frac{v}{c^2}\right)^2}$$

Contd.

Hence the Transformation equations for acceleration are given as

For observer in RF-1

$$a_{1x} = \frac{a_{2x}}{\gamma^3 \left[1 + \frac{v}{c^2} V_{2x}\right]^3} \quad \text{--- (13)}$$

$$a_{1y} = \left(a_{2y} - \frac{V_{2y}\, a_{2x}\, v}{c^2 + V_{2x} v}\right) \times \frac{1}{\gamma^2 \left(1 + V_{2x} \frac{v}{c^2}\right)^2} \quad \text{--- (14)}$$

$$a_{1z} = \left(a_{2z} - \frac{V_{2z}\, a_{2x}\, v}{c^2 + V_{2x} v}\right] \times \frac{1}{\gamma^2 \left(1 + V_{2x} \frac{v}{c^2}\right]^2} \quad \text{--- (15)}$$

For RF-2 observer

$$a_{2x} = \frac{a_{1x}}{\gamma 3 \left[1 - \frac{v}{c^2} V_{1x}\right]} \quad \text{--- (16)}$$

$$a_{2y} = \left(a_{1y} + \frac{V_{1y}\, a_{1x}\, v}{c^2 - V_{1x} v}\right] \times \frac{1}{\gamma^2 \left(1 - V_{1x} \frac{v}{c^2}\right]^2} \quad \text{--- (17)}$$

$$a_{2z} = a_{1z} + \frac{V_{1z}\, a_{1x}\, v}{c^2 - V_{1x} v}\Bigg] \times \frac{1}{\gamma^2 \left(1 - V_{1x} \frac{v}{c^2}\right)^2} \quad \text{--- (18)}$$