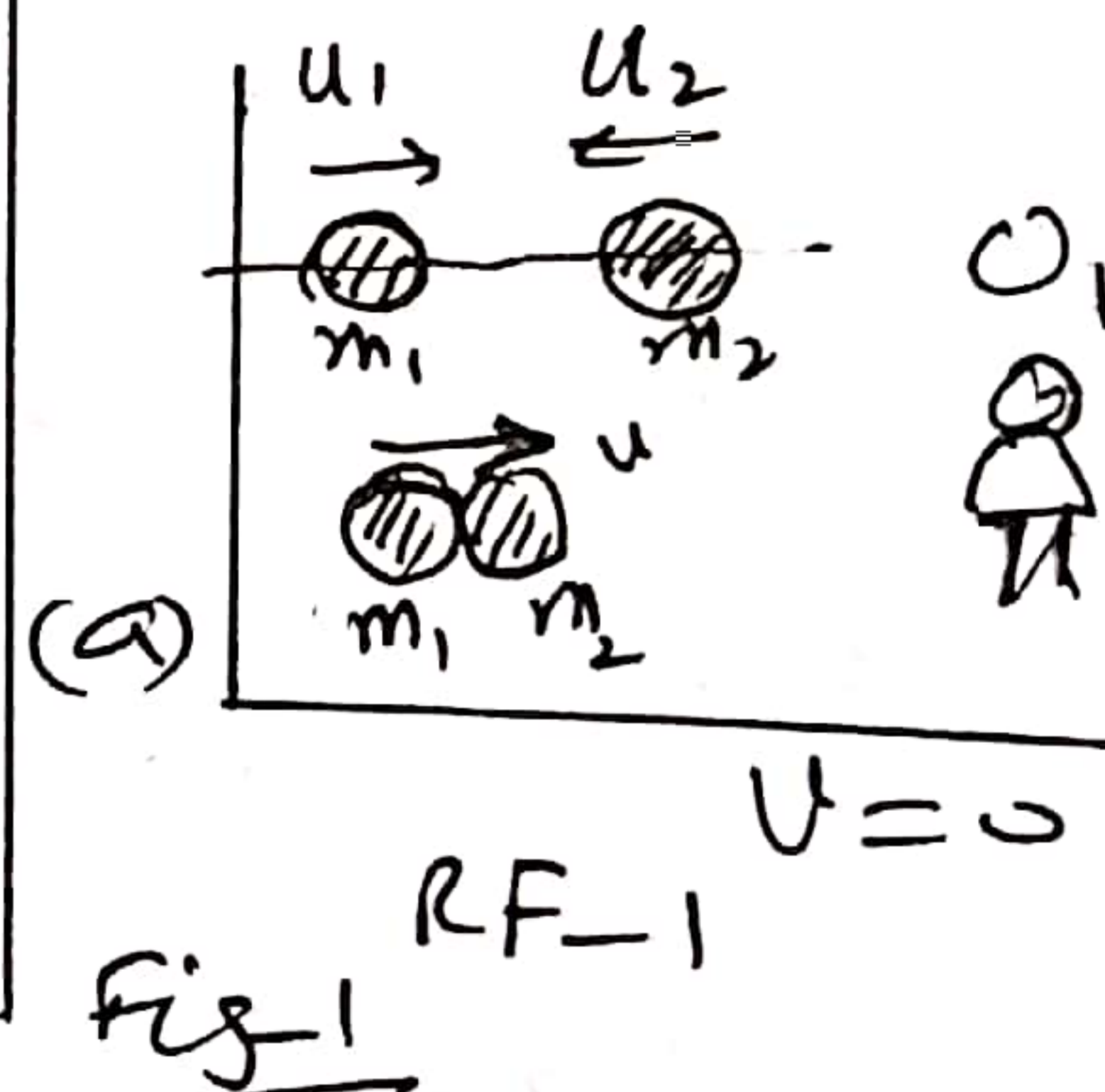
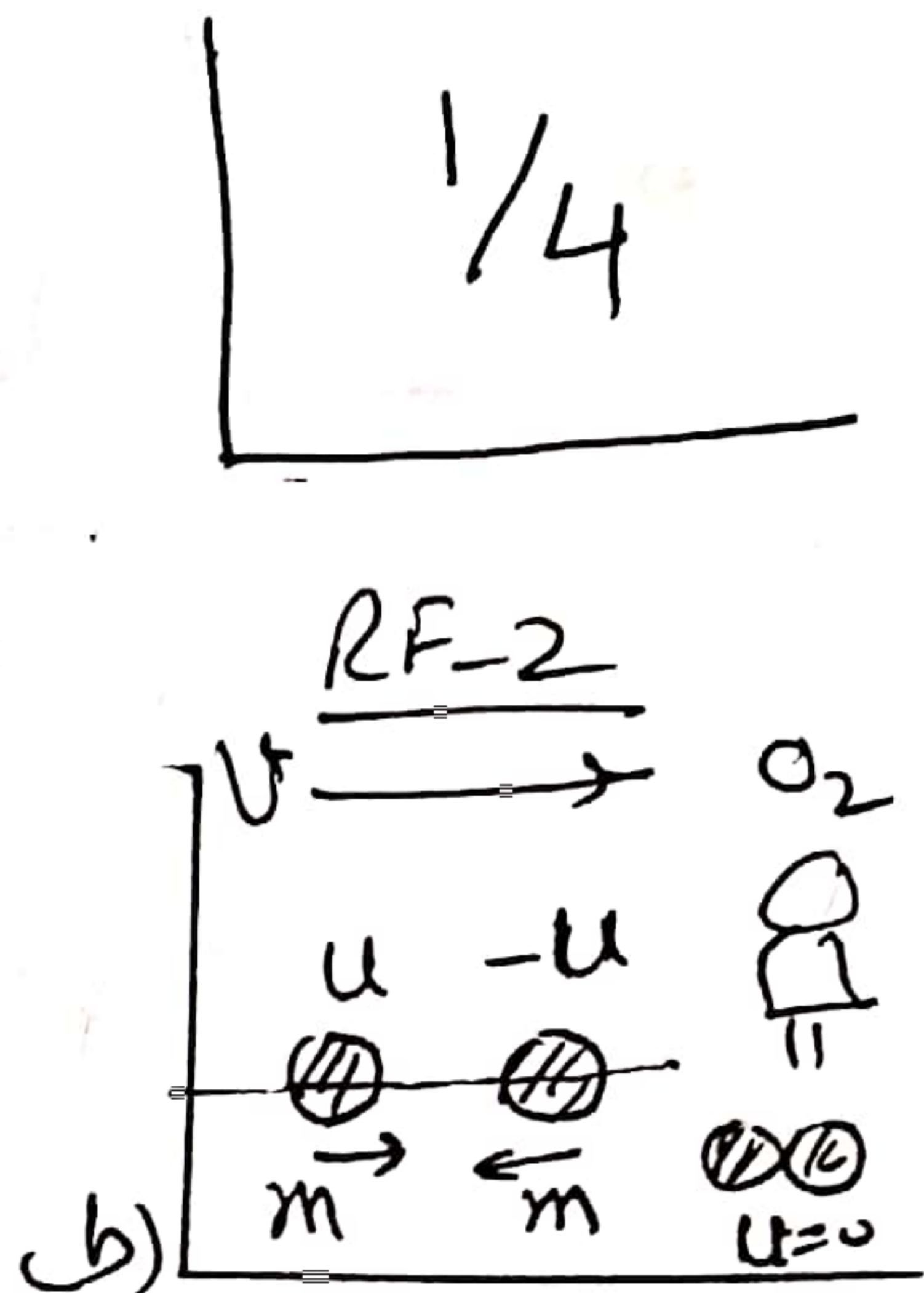


Transformation of mass:

Let us consider an observer O_2 in reference frame - 2 (RF-2). Let RF-2 be moving with velocity " U " which is uniform & comparable to the speed of light.

The observer O_2 observes an event occurring that two masses of same mass are moving towards each other with velocities u & $-u$. After head on collision both masses are at rest with $u=0$ as shown in Fig-1 (a) & (b), parts.

An observer " O_1 " in the "RF-1" watching this event happening in RF-2 with respect to his Reference frame RF-1 which is at rest with respect to RF-2. Let m_1 & m_2 be the masses & u_1 & u_2 be the velocities of masses m_1 & m_2 as observed by observer O_1 in RF-1.



Contd:

The momentum before and after collision as observed by observer O_1 is given as

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v \quad (1)$$

From the Transformation equations of Velocities we have

$$u_1 = \frac{u+v}{1 + \frac{uv}{c^2}} \quad (2)$$

$$u_2 = \frac{-u+v}{1 - \frac{uv}{c^2}} \quad (3)$$

Putting equations (2) & (3) in equation (1)

$$m_1 \left(\frac{u+v}{1 + \frac{uv}{c^2}} \right) + m_2 \left(\frac{-u+v}{1 - \frac{uv}{c^2}} \right) = m_1 v + m_2 v$$

Re-arranging the terms we get

$$m_1 \left[\left(\frac{u+v}{1 + \frac{uv}{c^2}} \right) - v \right] = m_2 \left[v - \left(\frac{-u+v}{1 - \frac{uv}{c^2}} \right) \right]$$

$$m_1 \left(\frac{u+v}{1 + \frac{uv}{c^2}} - v \left[\frac{1 + \frac{uv}{c^2}}{1 + \frac{uv}{c^2}} \right] \right) = m_2 \left(v \left[\frac{1 - \frac{uv}{c^2}}{1 - \frac{uv}{c^2}} \right] - \frac{-u+v}{1 - \frac{uv}{c^2}} \right)$$

$$m_1 \left(\frac{u+v - v - \frac{v^2}{c^2}}{1 + \frac{uv}{c^2}} \right) = m_2 \left(\frac{v - \frac{uv^2}{c^2} - v + u}{1 - \frac{uv}{c^2}} \right)$$

$$m_1 \left(\frac{u - \cancel{u} \left(\frac{v}{c} \right)^2}{1 + \frac{u v}{c^2}} \right) = m_2 \left(\frac{u - \cancel{u} \left(\frac{v}{c} \right)^2}{1 - \frac{u v}{c^2}} \right) \quad \left. \vphantom{\frac{m_1}{m_2}} \right| \quad \frac{3}{4}$$

$$\frac{m_1}{m_2} = \left(\frac{1 + \frac{u v}{c^2}}{1 - \frac{u v}{c^2}} \right) \quad \text{--- (4)}$$

Let us take $m_1 = m$ be the relativistic mass & $m_2 = m_0$ be the rest mass & $u = v$

$$\frac{m_1}{m_0} = \frac{m}{m_0} = \frac{1 + \frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}} = \frac{1 + \frac{v^2}{c^2}}{\left(1 + \frac{v^2}{c^2} - \frac{2v^2}{c^2} \right)} \quad \left. \vphantom{\frac{m}{m_0}} \right| \quad \begin{array}{l} \text{ii Hint} \\ (1 - \frac{v^2}{c^2}) \\ = (1 + \frac{v^2}{c^2} - \frac{2v^2}{c^2}) \end{array}$$

$$\frac{m}{m_0} = \frac{\left(1 + \frac{v^2}{c^2} \right)}{\left(\frac{1 + \frac{v^2}{c^2}}{c^2} \right) \left(1 - \frac{2v^2}{c^2} \right)} \quad \text{--- (5)}$$

$$\frac{m}{m_0} = \frac{1}{\left(1 - \frac{2v^2}{c^2} \right)} \quad \left(\frac{1 + \frac{v^2}{c^2}}{1 + \frac{v^2}{c^2}} \right)$$

Since we assumed $u = v$
So the velocity u of mass $m_1 = m$
 $u_1 = \frac{u + v}{1 + \frac{u v}{c^2}} = \left(\frac{2v}{1 + \frac{v^2}{c^2}} \right) \quad \text{--- (6)}$

Contd.

Putting " u_1 " value from equation (6) in equation (5) we get

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$$\frac{m}{m_0} = \left(\frac{1}{1 - u_1 \frac{v}{c^2}} \right) \quad \text{--- (7)}$$

Further from equation (6) we can get the following relation.

$$u_1 = \frac{2v}{1 + \frac{v^2}{c^2}}$$

$$u_1 \left(1 + \frac{v^2}{c^2} \right) = 2v$$

$$-2v + u_1 \frac{v^2}{c^2} = -u_1$$

$$\frac{u_1}{c^2} \left(-2v + u_1 \frac{v^2}{c^2} \right) = -\frac{u_1}{c^2}$$

~~adding~~ adding value '1' to the both sides of above equation

$$\therefore \left(1 - \frac{2vu_1}{c^2} + \frac{u_1 v^2}{c^4} \right) = 1 - \frac{u_1^2}{c^2}$$

$$\left(1 - \frac{u_1 v}{c^2} \right)^2 = 1 - \frac{u_1^2}{c^2}$$

$$\left(1 - \frac{u_1 v}{c^2} \right) = \sqrt{1 - \frac{u_1^2}{c^2}} \quad \text{--- (8)}$$

Putting in eqn (7) we get as $u_1 = v$

$$\frac{m}{m_0} = \frac{1}{\sqrt{1 - \frac{u_1^2}{c^2}}} \quad \left(m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \right) \quad \text{--- (9)}$$

Which shows that as relativistic speed increases.