Transformation of mass:

Let us consider an observer O_2 in reference frame-2 (RF-2). Let RF2 be moving with velocity "v" which is uniform & comparable to the speed of light. The observer O_2 observes an event occurring that two masses of same mass are moving towards each other with velocities u & $-u$. After headon collision both masses are at rest with $u=0$ as shown in Fig-1 (a) & (b), parts.

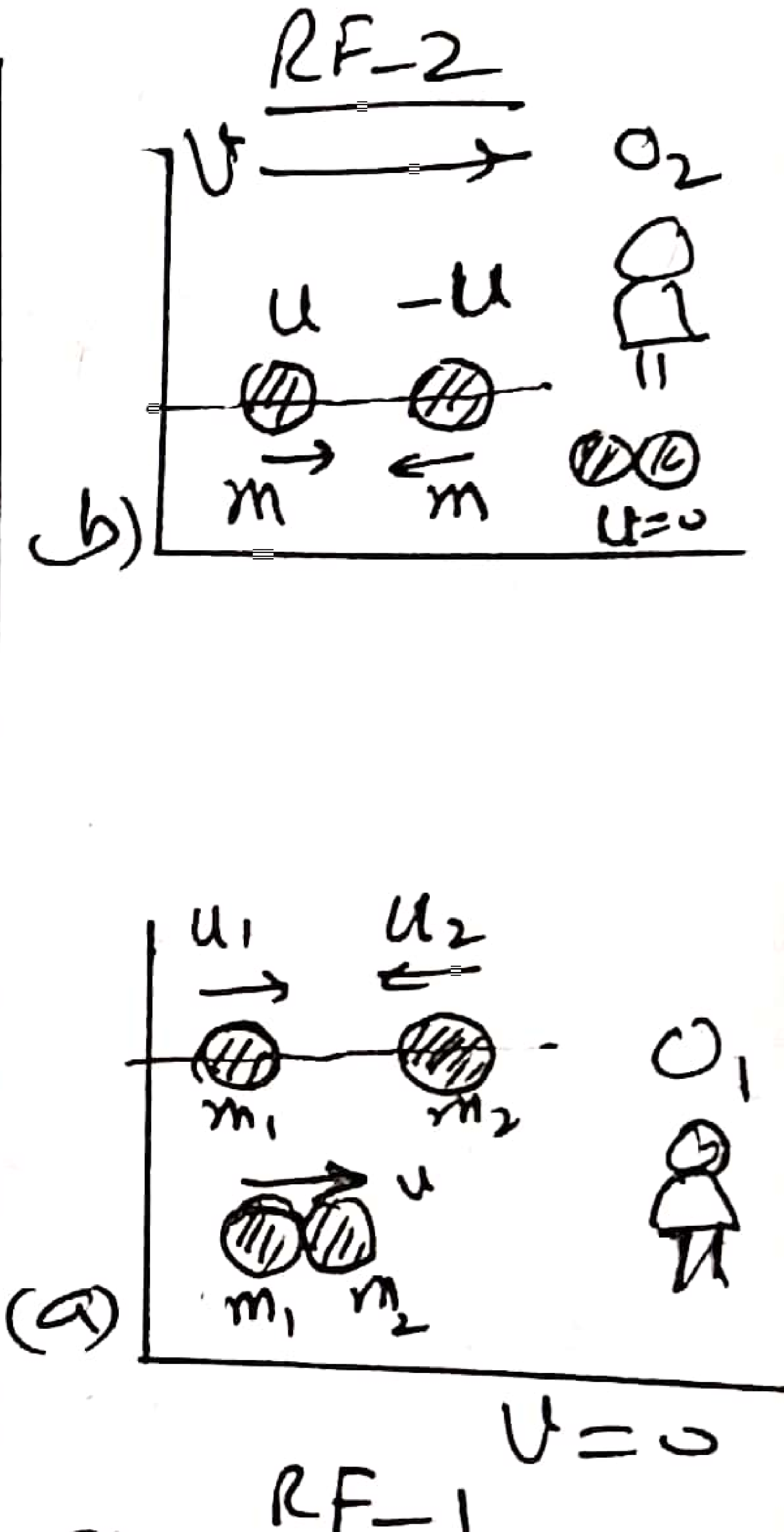

Fig-1

An observer O_1 in the RF-1 watching this event happening in RF-2 with respect to his Reference frame RF-1 which is at rest with respect to RF-2.

Let m_1 & m_2 be the masses & u_1 & u_2 be the velocities of masses m_1 & m_2 as observed by observer O_1 in RF-1.

Contd:

The momentum before and after collision as observed by Observer O_1 is given as

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v \text{ --- } (1)$$

From The Transformation equations of Velocities we have

$$u_1 = \frac{u+v}{1+\frac{uv}{c^2}} \text{ --- } (2)$$

$$u_2 = \frac{-u+v}{1-\frac{v}{c^2}u} \text{ --- } (3)$$

putting equations (2) & (3) in equation (1)

$$m_1\left(\frac{u+v}{1+\frac{v}{c^2}u}\right) + m_2\left(\frac{v-u}{1-\frac{v}{c^2}u}\right) = m_1 v + m_2 v$$

Re-arranging the terms we get

$$m_1\left[\left(\frac{u+v}{1+\frac{v}{c^2}u}\right) - v\right] = m_2\left[v - \left(\frac{v-u}{1-\frac{v}{c^2}u}\right)\right]$$

$$m_1\left[\frac{u+v}{1+\frac{uv}{c^2}} - v\left[1+\frac{uv}{c^2}\right]\right] = m_2\left[v\left(1-\frac{v}{c^2}u\right) - \frac{v-u}{1-\frac{v}{c^2}u}\right]$$

$$m_1\left[\frac{u+\cancel{v}-\cancel{v}-\frac{uv^2}{c^2}}{\left(1+\frac{uv}{c^2}\right)}\right] = m_2\left[\frac{\cancel{v}-\frac{uv^2}{c^2}-\cancel{v}+u}{\left(1-\frac{uv}{c^2}\right)}\right]$$

Contd.

$$m_1\left[\frac{u-\cancel{u(v/c)}}{1+\frac{uv}{c^2}}\right]^2 = m_2\left[\frac{u-\cancel{u\frac{v}{c}}}{1-\frac{uv}{c^2}}\right]^2$$

$$\frac{m_1}{m_2} = \left(\frac{1+uv/c^2}{1-uv/c^2}\right) \quad \text{—— (4)}$$

Let us take $m_1 = m$ be the relativistic mass & $m_2 = m_0$ be the rest mass & $u = v$

$$\frac{m_1}{m_0} = \frac{m}{m_0} = \frac{1+v^2/c^2}{1-v^2/c^2} = \frac{1+v^2/c^2}{\left(1+\frac{v^2}{c^2}-\frac{2v^2}{c^2}\right)}$$

$\because$ Hint

$$\left(1-v^2/c^2\right) = \left(1+\frac{v^2}{c^2}-\frac{2v^2}{c^2}\right)$$

$$\frac{m}{m_0} = \frac{\cancel{(1+v^2/c^2)}}{\cancel{\left(1+\frac{v^2}{c^2}\right)}\left(1-\frac{2v^2/c^2}{1+v^2/c^2}\right)}$$

$$\frac{m}{m_0} = \frac{1}{\left(1-\frac{2v^2/c^2}{1+v^2/c^2}\right)} \quad \text{—— (5)}$$

Since we assumed $u = v$

So the velocity u_1 of mass $m_1 = m$

$$u_1 = \frac{u+v}{1+\frac{uv}{c^2}} = \left(\frac{2v}{1+v^2/c^2}\right) \quad \text{—— (6)}$$

Contd.

putting "u_1" value from equation (6) in equation (5) we get

$$\frac{m}{m_o} = \left(\frac{1}{1 - u_1 v/c^2} \right) \quad —(7)$$

Further from equation (6) we can get the following relation.

$$u_1 = \frac{2v}{1 + v^2/c^2}$$

$$u_1(1 + v^2/c^2) = 2v$$

$$-2v + u_1 v^2/c^2 = -u_1$$

$$\frac{u_1}{c^2}(-2v + u_1 v^2/c^2) = -u_1 \frac{u_1}{c^2}$$

adding value '1' to the both sides of above equation

$$\left(1 - \frac{2v u_1}{c^2} + \frac{u_1 v^2}{c^4}\right) = 1 - u_1^2/c^2$$

$$\left(1 - \frac{u_1 v}{c^2}\right)^2 = 1 - u_1^2/c^2$$

$$\left(1 - \frac{u_1 v}{c^2}\right) = \sqrt{1 - u_1^2/c^2} \quad —(8)$$

putting in equat (7) we get as $u_1 \approx v$

$$\frac{m}{m_o} = \frac{1}{\sqrt{1 - u_1^2/c^2}}, \quad m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \quad —(9)$$

Which Shows that at relativistic Speed, mass increases.