Transformation of momentum and energy: (a) Transformation of mass:

Let us consider the momentum of a mass "m" moving at uniform velocity "u" in RF-2 and let the p_1 be the momentum observed by the observer "O_1"

$$p_1 = m_1 v_1 \quad \text{——— (1)}$$

In three coordinates it is given as

$$p_{1x} = m_1 u_{1x}, \quad p_{1y} = m_1 v_{1y} \quad \& \quad p_{2z} = m_2 u_{2z}$$

$$p_{1x} = m_1 v_{1x} \qquad \text{putting } u_{1x} = \frac{v_{2x} + v}{1 + v_{2x} v/c^2}$$

$$\& \; {}^{*}m_1 = m_2 \left(1 + v_{2x}\frac{v}{c^2}\right)\gamma$$

$$p_{1x} = m_2 \gamma \left[1 + \cancel{v_{2x}\frac{v}{c^2}}\right]\left[\frac{v_{2x} + v}{\cancel{1 + v_{2x} v/c^2}}\right]$$

$$p_{1x} = \gamma\,(m_2 v_{2x} + m_2 v) \text{ —— (2)} \qquad \text{as } E_2 = m_2 c^2$$

$$\therefore \; p_{2x} = m_2 v_{2x} \quad \& \text{ let } m_2 = \frac{E_2}{c^2}$$

$$p_{1x} = \gamma\left[p_{2x} + v\frac{E_2}{c^2}\right] \text{ —— (3)}$$

$$p_{1y} = p_{2y}$$

$$p_{1z} = p_{2z}$$

Contd.

Hence the momentum Transformation equations given as

RF-1

$$p_{1x} = \gamma\left(p_{2x} + v\frac{\mathcal{E}_2}{c^2}\right)$$

$$p_{1y} = p_{2y}$$

$$p_{1z} = p_{2z}$$

RF-2

$$p_{2x} = \gamma\left(p_{1x} - v\frac{\mathcal{E}_2}{c^2}\right)$$

$$p_{2y} = p_{1y}$$

$$p_{2z} = p_{1z}$$

(b) Transformation of Energy:

The energy equation for the mass "m_1" is given as

$$\mathcal{E}_1 = m_1 c^2 \quad \text{---(1)}$$

$$m_1 = m_2 \gamma \left[1 + V_{2x} v / c^2\right]$$

$$\mathcal{E}_1 = \gamma\left[1 + V_{2x}\frac{v}{c^2}\right] m_2 c^2 \quad \text{---(2)}$$

$$\mathcal{E}_1 = \gamma\left[m_2 c^2 + m_2 V_{2x} \frac{v}{\cancel{c^2}} \times \cancel{c^2}\right]$$

$$\mathcal{E}_1 = \gamma\left[\mathcal{E}_2 + v p_{2x}\right] \quad \text{---(3)}$$

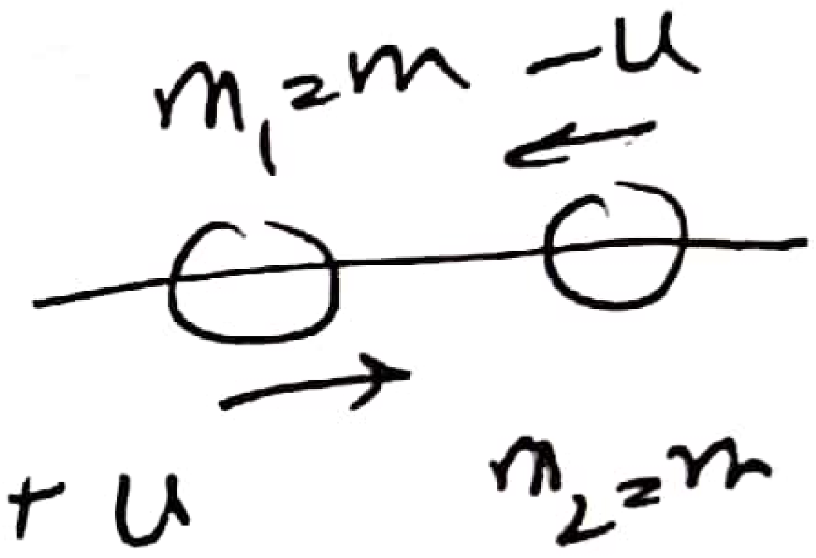
$$\mathcal{E}_2 = \gamma\left[\mathcal{E}_1 - v p_{1x}\right] \quad \text{---(4)}$$

$$* \quad m_1 = \frac{m_2\left(1 + v_{2x} u/c^2\right)}{\sqrt{1 - u^2/c^2}}$$

Recalling the momentum relations for m_1 & m_2 we had

$$m_1 = m_2 \left(\frac{1 + u\, v/c^2}{1 - u\, v/c^2} \right)$$

Where $+u$ was the velocity of mass "m_1" & $-u$ for m_2



$$m_1 = m_2 \frac{\left[1 + v_{2x} u/c^2\right]}{\left(1 - v^2/c^2\right)}.$$

let $+u = v_{2x}$

& $-u = -v$