Transformation of "Force":

Let us now consider the transformation of "Force" if a reference frame is moving with the relativistic speed.

Newtons Force is given as

$$\bar{F} = m\bar{a}$$

Since $\bar{a} = \frac{d\bar{v}}{dt}$

& at relativistic Velocity "m" is not a constant quantity so

$$\bar{F} = \frac{d(m.\bar{v})}{dt} = \frac{d\bar{P}}{dt} \quad \text{---(1)}$$

Let us consider $\bar{F}_1$ force is observed in RF-1 which is at rest & $\bar{F}_2$ force in RF-2 which is moving with constant velocity "v".

$$\bar{F}_1 = F_{x_1}\hat{i} + F_{y_1}\hat{j} + F_{z_1}\hat{k} \quad \text{---(2)}$$

$$\& \quad \bar{F}_2 = F_{x_2}\hat{i} + F_{y_2}\hat{j} + F_{z_2}\hat{k} \quad \text{---(3)}$$

Taking magnitude $F_{x_1} = \frac{dP_{1x}}{dt_1}$ ---(4)

Since $P_{1x} = \left(\gamma P_{2x} + v\frac{E_2}{c^2}\right)$ ---(4)

We can write eq(4), if we multiply & divide "dt_2"

$$F_{x_1} = \frac{dP_{1x}}{dt_2} \times \frac{dt_2}{dt_1} \quad \text{---(5)}$$

also we have the relation

$$\frac{dt_2}{dt_1} = \left(\frac{1}{\gamma\left[1 + V_{2x} \frac{v}{c^2}\right]} \right) \quad -(6)$$

putting the respective terms from (4) & (6) in equation (5) we have

$$F_{x_1} = \frac{d}{dt_2}\left[\cancel{\gamma} P_{2x} + \cancel{\gamma} v \frac{\mathcal{E}_2}{c^2}\right] \times \frac{1}{\cancel{\gamma}\left[1 + V_{2x}\frac{v}{c^2}\right)}$$

$$F_{x_1} = \left[\frac{dP_{2x}}{dt_2} + \frac{v}{c^2}\frac{d\mathcal{E}_2}{dt_2} \right] \times \frac{1}{\left(1 + V_{2x} \frac{v}{c^2}\right)} \quad -(7)$$

* Hint

$\mathcal{E} = F.x$

$\frac{d\mathcal{E}}{dt} = F\frac{dx}{dt}$

$\frac{d\mathcal{E}}{dt} = F.v$

If F = const

But * $\frac{d\mathcal{E}_2}{dt_2} = F_{x_2} V_{2x} + F_{y_2} V_{2y} + F_{z_2} V_{2z} \quad -(8)$

putting (8) in eq (7) where, $F_{x_2} = \frac{dP_{2x}}{dt_2}$

$$F_{x_1} = \frac{F_{x_2} + \frac{v}{c^2}\left(F_{x_2}V_{2x} + F_{y_2}V_{2y} + F_{z_2}V_{2z}\right)}{\left(1 + V_{2x} v/c^2\right)}$$

$$= \frac{F_{x_2} + \frac{v}{c^2} F_{x_2} V_{2x} + v/c^2 \left(F_{y_2} V_{2y} + F_{z_2} V_{2z}\right)}{\left(1 + V_{2x} v/c^2\right)}$$

$$= \frac{F_{x_2}\left(1 + v/c^2\, V_{2x}\right) + v/c^2\left(F_{y_2} V_{2y} + F_{z_2} V_{2z}\right)}{\left(1 + V_{2x} v/c^2\right)}$$

$$F_{x_1} = \frac{\cancel{\left(1 + v/c^2\, V_{2x}\right)}\left(F_{x_2} + v/c^2 \frac{\left(F_{y_2}V_{2y} + F_{z_2}V_{2z}\right)}{1 + V_{2x} v/c^2}\right)}{\cancel{\left(1 + V_{2x} v/c^2\right)}}$$

contd:

$$F_{x_1} = F_{x_2} + \frac{\frac{v}{\cancel{c^2}}(F_{y_2}V_{2y} + F_{z_2}V_{2z})}{(c^2 + vV_{2x})/\cancel{c^2}}$$

$$F_{x_1} = F_{x_2} + \frac{v}{c^2 + vV_{2x}}\{F_{y_2}V_{2y} + F_{z_2}V_{2z}\} \quad \text{—(9)}$$

$v_y = 0 \quad v \to v_x$

Similarly we can find $(\gamma \geq 1)$, $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

$$F_{y_1} = \frac{dP_{1y}}{dt_1} = \frac{dP_{1y}}{dt_1} \times \frac{dt_2}{dt_2} \qquad P_{1y} = P_{2y}$$

$E_2 = \text{const}$ as No change along y axis. The change is along x only. $\frac{dE_2}{dt_2} = 0$

$$F_{y_1} = \frac{dP_{1y}}{dt_2} \times \left(\frac{dt_2}{dt_1}\right) \quad \text{—(10)}$$

$$= \frac{d}{dt_2}(P_{2y}) \times \left(\frac{1}{\gamma[1 + V_{2x}v/c^2]}\right)$$

$$F_{y_1} = \frac{F_{y_2}}{\gamma(1 + V_{2x}v/c^2)} \quad \text{—(11)}$$

$$\& \; F_{z_1} = \frac{F_{z_2}}{\gamma(1 + V_{2x}v/c^2)} \quad \text{—(12)}$$

For Reference Frame – 2

$$F_{x_2} = F_{x_1} - \frac{v}{(c^2 - V_{1x}v)}(U_1 F_{y_1} + U_{1x}F_{z_1}) \quad \text{—(13)}$$

$$F_{y_2} = \frac{F_{y_1}}{\gamma[1 - V_{1x}v/c^2)} \quad \text{—(14)}$$

$$\& \; F_{z_2} = \frac{F_{z_1}}{\gamma[1 - V_{1x}v/c^2]} \quad \text{—(15)}$$