The four Vector "$\bar{r}$":

An event such as emission of a flash light is characterized by its position (x, y, z) & by time "t" at which it occurs.

Let us examine these four variables more closely.

We shall find that they can be expressed by a single equation that is invariant under the Lorentz Transformation and that proves to be useful for discovering other invariant quantities in mechanical & EM-phenomenon.

Under Galilean Transformation the distance between two points (x_a, y_a, z_a) & (x_b, y_b, z_b) is invariant

$$r_{ab}^2 = (x_{a_1} - x_{b_1})^2 + (y_{a_1} - y_{b_1})^2 + (z_{a_1} - z_{b_1})^2 \quad (1)$$

$$= (x_{a_2} - x_{b_2})^2 + (y_{a_2} - y_{b_2})^2 + (z_{a_2} - z_{b_2})^2 \quad (2)$$

Note that the word invariant is not synonymous with the word constant. In fact if the points "a's" were fixed in RF-1 & b's were fix in RF-2, then r_{ab} would be function of time.
The distance r_{ab} is said to be invarient because it is given correctly by the same expression in both frames.

Contd:

related by the equation given as

$$x_1^2+y_1^2+z_1^2-c^2t_1^2 = x_2^2+y_2^2+z_2^2-c^2t_2^2 \quad (5)$$

$$r_1^2-c^2t_1^2 = r_2^2-c^2t_2^2 \quad (6)$$

we can check this quite easily the quantity $r^2-c^2t^2$ is, therefore invariant under the Lorentz Transformation.

Now this property of (x, y, z, t) follows directly from Lorentz Transformation "then for any set of four quantities that transform like (x, y, z, t) we have a corresponding invariant quantity."

Such sets of four quantities are called Four-Vectors. By analogy with three dimensional geometry we specify the coordinates of event as being (x, y, z, jct) where $j=\sqrt{-1}$ & the magnitude of the four dimension distance between the event and the origin is the square root of the sum of the squares of the components.

$$\bar{r} = (r, jct) \quad (7)$$

$|\bar{r}| = \sqrt{x^2+y^2+z^2-c^2t^2}$ is called the magnitude of four-vector. $(\bar{r})^2 = r^2-c^2t^2 \quad (8)$

Hence the quantity is invariant.

Now we can easily show that with Lorentz Transformation V_{ab} is not invariant. However there does exist a corresponding quantity that is invariant under a Lorentz Transformation.

Imagine that a flash of light is emitted at point at. O_1 at the moment when two regions coincide The light propagates in all directions with velocity 'c' in both systems. i.e

$$\frac{x_1^2+y_1^2+z_1^2}{t_1^2} = \frac{x_2^2+y_2^2+z_2^2}{t_2^2} = c^2 \quad (3)$$

Where (x_1, y_1, z_1) & (x_2, y_2, z_2) are the coordinates of RF-1 & RF-2 respectively & t_1, t_2 are the respective time intervals of RF1 & RF-2.

The light arrives at (x_1, y_1, z_1) at time t_1 & at point (x_2, y_2, z_2) at time t_2

Thus This can be written as

$$x_1^2+y_1^2+z_1^2-c^2t_1^2 = x_2^2+y_2^2+z_2^2-c^2t_2^2 = 0 \quad (4)$$

more generally the coordinates (x_1, y_1, z_1, t_1) & (x_2, y_2, z_2, t_2) for a single event are

Contd