Dispersion:

The EM-waves when propagate through the region of space where there is no charge or current Maxwell's equations are given as:

$$\text{(i)}\quad \bar{\nabla}\cdot\bar{E}=0 \qquad\qquad \text{(iii)}\quad \bar{\nabla}\times\bar{E}=-\frac{\partial\bar{B}}{\partial t}$$

$$\text{(ii)}\quad \bar{\nabla}\cdot\bar{B}=0 \qquad\qquad \text{(iv)}\quad \bar{\nabla}\times\bar{B}=\epsilon_0\mu_0\frac{\partial\bar{E}}{\partial t} \qquad \text{—(1)}$$

Now if the EM-waves travel inside a medium but in regions where there is no free charge or free current then the Maxwell's equations become

$$\text{(i)}\quad \bar{\nabla}\cdot\bar{D}=0 \qquad\qquad \text{(iii)}\quad \bar{\nabla}\times\bar{E}=-\frac{\partial\bar{B}}{\partial t}$$

$$\bar{\nabla}\cdot\bar{B}=0 \qquad\qquad \text{(iv)}\quad \bar{\nabla}\times\bar{H}=\frac{\partial\bar{D}}{\partial t} \qquad \text{—(2)}$$

We consider in the above case the medium to be Linear & homogenous so that ϵ & μ do not vary from point to point.

For a linear & homogenous medium the Maxwell's equations are given as

Contd:

(i) $\bar{\nabla}\cdot\bar{E} = 0$ (iii) $\bar{\nabla}\times\bar{E} = -\frac{\partial \bar{B}}{\partial t}$

(ii) $\bar{\nabla}\cdot\bar{B} = 0$ (iv) $\bar{\nabla}\times\bar{B} = \epsilon\mu\frac{\partial \bar{E}}{\partial t}$ —(3)

Maxwell's equation in (3) differes from equation (1) only in the replacement of $\epsilon_0\mu_0$ by $\epsilon\mu$. Evidently when EM-wave propagate through a linear homogenous medium at a speed

$$v = \frac{1}{\sqrt{\epsilon\mu}} \quad \text{—(4)}$$

for optically transparent media μ_r is typically close to "1" & ϵ_r is greater than "1"

$$v = \frac{1}{\sqrt{\mu_0\epsilon_0\epsilon_r}} \quad \text{—(5)}$$

evidently then the light should travel slower through matter than free space a fact that is of course well known from optics $v = \frac{c}{n}$ —(6)

Where 'v' is the speed of the wave, "c" is the speed of light & "n" is the refractive index

It follows that "n" the refractive index is related to the electric and magnetic properties of the material as $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ & $v = \frac{1}{\sqrt{\epsilon \mu}}$

putting in eq (6),

$$n = \frac{c}{v} = \sqrt{\frac{\epsilon \mu}{\epsilon_0 \mu_0}} \quad \text{——(7)}$$

Also $n = \frac{c}{v}$, where $v = f\lambda \Rightarrow \lambda = \frac{v}{f}$

$$\left(n \propto \frac{1}{\lambda}\right)$$

The index of refraction typically varies some what according to the wave length (color) of incident light (white light). That's what accounts for the dispersion of light by a prism, or a droplet of water in the formation of rainbow.

Hence the Dispersion is the phenomenon in which EM-waves are affected by the EM-properties of medium.

Dispersion:

Frequency dependence of ϵ, μ & σ

We have observed that the propagation of EM-Waves through matter is governed by three properties of material which we took to be constants, the permitivity "ϵ", the permeability "μ" & the conductivity "σ".

Actually each of these parameters depends to some extent on the frequency of waves. Indeed if ϵ & μ were true constant, then the speed of wave in a non-conducti

$$v = \frac{1}{\sqrt{\epsilon\mu}} \qquad (1)$$

& the index of refraction $n = \frac{c}{v}$, be constant

$$v = \frac{\omega}{k} \Rightarrow n = \frac{c}{\omega}k \Rightarrow n = \frac{c}{\omega} \times \frac{2\pi}{\lambda}$$

$$n = \frac{c}{2\pi f} \times \frac{2\pi}{\lambda} = \frac{c}{f} \times \left(\frac{1}{\lambda}\right)$$

$$n \propto \frac{1}{\lambda}$$

Since it is well known from optics that "n" is function of "ω" and also that of "λ".

Contd.

A graph of a typical glass prism is shown in fig – 1, n vs λ

Thus a prism bends blue light more sharply than red and spreads white light in to spectrum of colors.

This phenomenon in which white light disperse in to various colors, is called "dispersion". By extension, whenever the speed of wave varies with its frequency in a medium, then such a supporting medium is called a dispersive medium