

## Dispersion in Conductors:

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The free electrons in conductors are not bound to any particular atom or molecule, nevertheless, essentially the same model i.e. electron model can be applied to them.

If we set the binding force equal to zero and consider that the damping factor " $\gamma$ " is attributable (consequence) to the cumulative effect of many collisions is relatively large. The equation of motion takes the form as

$$m \frac{d^2 \vec{y}}{dt^2} = F_{\text{damp}} + F_{\text{driving}}$$

$$\text{At } F_{\text{damp}} = -m \kappa \frac{d\vec{y}}{dt}, \quad F_{\text{drive}} = \frac{q}{m} \vec{E}_0 e^{-i\omega t}$$

we have

$$\frac{d^2 \vec{y}}{dt^2} + \gamma \frac{d\vec{y}}{dt} = \frac{q}{m} \vec{E}_0 e^{-i\omega t} \quad (1)$$

$$\text{Since } \vec{y}_{\pm 1} = \vec{y}_0 e^{-i\omega t} \quad (2)$$

$$\frac{d\vec{y}_{\pm 1}}{dt} = -i\omega \vec{y}_0 e^{-i\omega t} = -i\omega \vec{y}_{\pm 1}$$

$$\text{and } \frac{d^2 \tilde{y}}{dt^2} = -\omega^2 \tilde{y}_0 e^{-i\omega t}$$

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Putting in eqn: (1), we have

$$\tilde{y}_0 = \frac{-q/m}{(\omega^2 - i\gamma\omega)} \tilde{E}_0 \quad (3)$$

This time we are not interested in polarization associated with the electrons but rather the current they generate

$$J = N f q \frac{dy}{dt} = (N f q v), \quad N = \frac{n}{V}$$

where  $f$  is the number of free electrons per unit volume.

$$J = I/A$$

$$I = q/t$$

Evidently  $J$  is the real part  $\tilde{J}$  which is complex. The complex  $\tilde{J}$  is of

the form 
$$\tilde{J} = N f q \frac{d\tilde{y}}{dt} \quad (4)$$

From eqn (2)

$$\frac{d\tilde{y}}{dt} = \left( \frac{q/m}{\gamma - i\omega} \right) \tilde{E} \quad \text{put in (4)}$$

$$\tilde{J} = N f q \left( \frac{q/m}{\gamma - i\omega} \right) \tilde{E} \quad (5)$$

Contd:

The imaginary term of  $\epsilon(\omega)$  in the denominator means  $\bar{J}$  is out of phase with  $\bar{E}$  i.e. current is not proportional to the field. Which means this material does not strictly speaking, obey Ohm's Law.

However the fact that the complex current is proportional to the complex field which suggests us to introduce a complex conductivity

$$\bar{J} = \tilde{\sigma}_c \bar{E} \quad \text{--- (6)}$$

Putting  $\bar{J}$  from equation (5) in (6), we have

$$\tilde{\sigma}_c = \frac{N f q^2 / m}{\gamma - i\omega} \quad \text{--- (7)}$$

But for sufficiently large frequencies (the phase difference between  $\bar{J}$  &  $\bar{E}$  can not be ignored. In fact the current can not keep up when the driving force oscillates too rapidly.

So naturally, all this affects the behaviour of EM-Waves in a conductor at high frequencies

Contd.

lets take

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$$\tilde{\sigma}_c = \sigma_R + i\sigma_I \quad \text{--- (8)} \quad \left[ \begin{array}{l} 4 \\ 14 \end{array} \right]$$

rationalizing equation (7) we get

$$\tilde{\sigma}_c = \frac{\gamma N f q^2 / m}{(\gamma^2 + \omega^2)} + i \frac{N \omega f q^2 / m}{(\gamma^2 + \omega^2)} \quad \text{--- (9)}$$

Comparing Real & imaginary parts

of equations (8) & (9)

$$\sigma_R = \left( \frac{\gamma N f q^2 / m}{\gamma^2 + \omega^2} \right)$$

$$\sigma_I = \left( \frac{\omega N f q^2 / m}{\gamma^2 + \omega^2} \right)$$

Hence for real  $\vec{J}$

$$\vec{J} = \sigma_R \vec{E}$$

$$\vec{J} = \left( \frac{\gamma N f q^2 / m}{\gamma^2 + \omega^2} \right) \vec{E} \quad \text{--- (10)}$$

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