## Dispersion in dilute medium: (Plasma)

Free electron model in conductors lead us to the complex conductivity which is affected at high frequencies

$$\tilde{\sigma}_c = \frac{N f q^2/m}{\gamma - i\omega} \quad \text{---} (1)$$

$$\tilde{J} = \tilde{\sigma}_c \tilde{E} \quad \text{---} (2)$$

The wave number, $K$, is given a

$$K^2 = \epsilon\mu\omega^2 + i\mu\sigma\omega \quad \text{---} (3)$$

Now to consider dilute plasma (highly ionized gas) the damping is neglegible & the conductivity $\sigma$ is purely imaginary i.e $\gamma = 0$

$$\tilde{\sigma} = -\frac{N f q^2/m}{i\omega} = i\left(\frac{N f q^2/m}{\omega}\right) \quad \text{---} (4)$$

More over $\mu \simeq \mu_0$ & $\epsilon = \epsilon_0$ in plasma then equation (3) is given as

Contd

$$K^2 = \mu_0 \epsilon_0 \omega^2 + i \mu_0 \sigma \omega$$

putting $\tilde{\sigma} = \dfrac{i N f q^2/m}{\omega}$

$$K^2 = \mu_0 \epsilon_0 \omega^2 + i\left(\frac{i N f q^2/m}{\cancel{\omega}}\right) \cancel{\omega} \mu_0$$

$$K^2 = \epsilon_0 \mu_0 \omega^2 - \frac{N f q^2 \mu_0}{m}$$

$$K^2 = \epsilon_0 \mu_0 \left[ \omega^2 - \omega_p^2 \right] \text{ —— (5)}$$

$$K^2 = \frac{1}{c^2} \left[ \omega^2 - \omega_p^2 \right] \text{ —— (6)}$$

Where $c = \dfrac{1}{\sqrt{\epsilon_0 \mu_0}}$ & $\omega_p = q\sqrt{\dfrac{Nf}{m \epsilon_0}}$

Where $\omega_p$ is called the plasma frequency. For frequencies higher than $\omega_p$ ie $\omega > \omega_p$ The wave number is real as $K = \dfrac{1}{c}\sqrt{\omega^2 - \omega_p^2}$

& waves propagate without attenuation at a wave speed.

Speed of the wave $v = \dfrac{\omega}{K}$

Contd

Since $k = \frac{\omega}{c}\sqrt{1-\left(\frac{\omega_p}{\omega}\right)^2}$

So $v = \frac{\omega}{\omega/c}\sqrt{1-\left(\frac{\omega_p}{\omega}\right)^2}$

$$v = c\sqrt{1-\left(\frac{\omega_p}{\omega}\right)^2} \quad \text{——} (7)$$

$$n = \frac{c}{v} = 1\Big/\sqrt{1-\left(\frac{\omega_p}{\omega}\right)^2} \quad \text{——} (8)$$

Equation (8) gives the relation for the index of refraction in plasma.

For frequencies below $\omega_p$ i.e $\omega < \omega_p$ on the other hand the $k$ is purely imaginary

$$\tilde{E}_{(x,t)} = \tilde{E}_0\, e^{i(kx-\omega t)}$$

for $\omega < \omega_p$

$$k = \frac{\omega}{c}\left(i\sqrt{\left(\frac{\omega_p}{\omega}\right)^2-1}\right) \quad \text{——} (9)$$

$$n = \frac{c}{v} = -i\frac{1}{\sqrt{\left(\frac{\omega_p}{\omega}\right)^2-1}} \quad \text{——} (10)$$

Accordingly the plasma is opaque to wave frequency less than $\omega_p$ ie $\omega < \omega_p \rightarrow$ plasma is opaque

& transparent to those above $\omega_p$

ie $\omega > \omega_p \rightarrow$ plasma is transparent