EMWG-S1 **Electromagnetic Wave Guides:**

So far we have studied the propagation of plane EM-waves in an infinite unbounded region in vacuum and other media.

We shall now study the propagation by means of guided waves, that is waves, that are guided along over a conducting or dielectric surface such as two parallel conducting planes, rectangular and cylinderical hollow conducting tubes of any uniform crossection which can support EM-waves.

A wave guide is a hollow conducting tube (metallic) of uniform crossection used for transmitting EM-waves by successive reflection from the inner walls of the tube.

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EM-Wave propagation between two parallel Conducting planes.

Let us consider the EM-wave propagation between two parallel perfectly conducting planes of infinite extent in the y & z directions. The two planes are separated by a distance "a" along the x-axis as shown in Fig-1

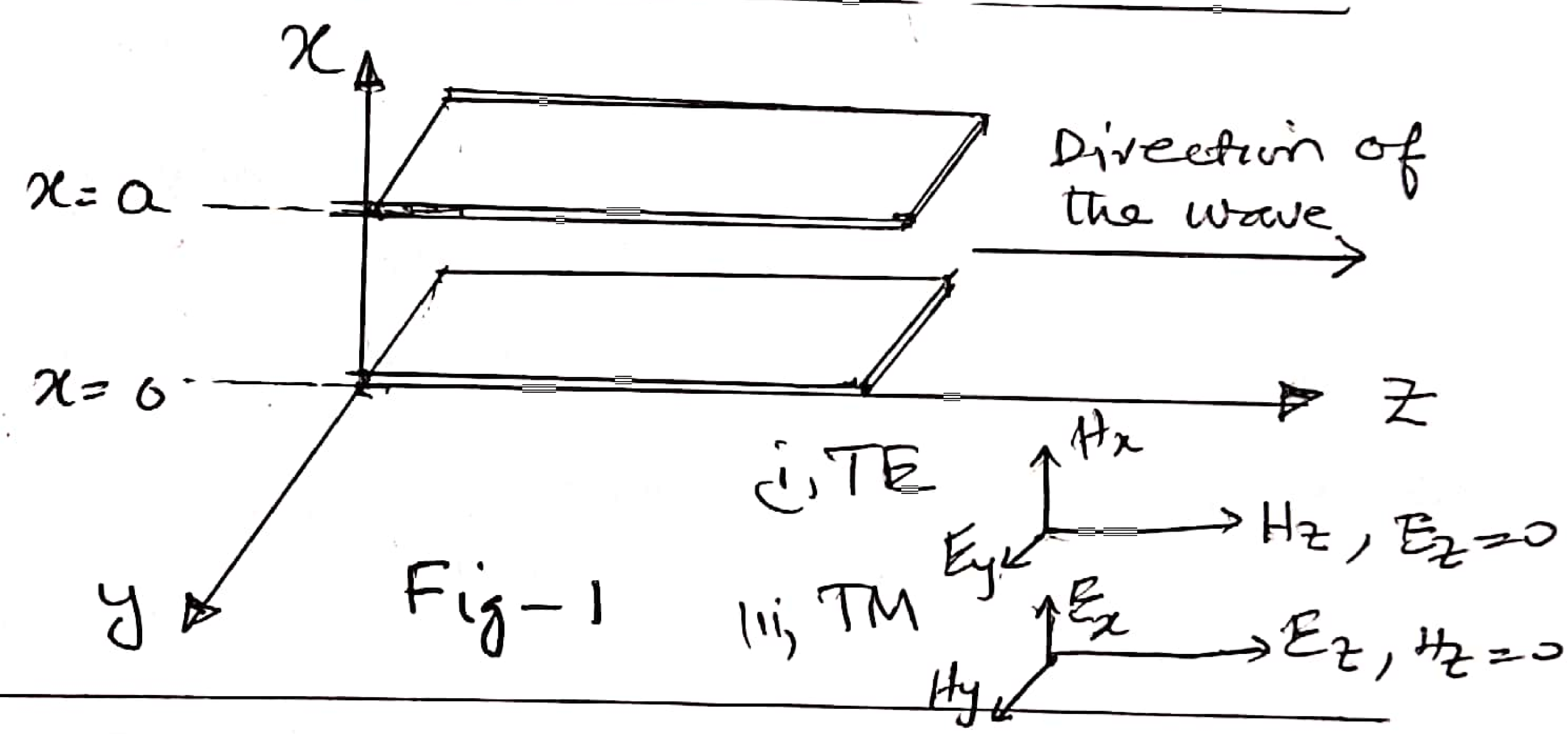

Fig-1

Since the planes have been assumed as perfectly conducting any component of $\bar{E}$ parallel to the surface must be zero. Which means that Electric field must always be perpendicular to the surface of the conductors.

The Magnetic field being perpendicular to the Electric field must be parallel to the surface of the conductors

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Thus $E_t = 0$

& $H_n = 0$

Where E_t is tangential compent and H_n is the normal component of $\bar{E}$ & H fields respectively.

Hence Maxwell's equations subject to these boundary conditions at the surface of the conductors and can determine the configuration of EM-fields in the space.

For non-conducting region between the planes i.e air the Maxwell's equations are of the form:

$$\bar{\nabla} \times \bar{H} = \epsilon \frac{\partial \bar{E}}{\partial t} \quad \text{——— (1)}$$

$$\bar{\nabla} \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t} \quad \text{——— (2)}$$

Assuming the $\bar{E}$ & $\bar{H}$ fields propagate as sinusoidal variations as $\bar{E} = \bar{E}_o e^{j\omega t}$ & $\bar{H} = \bar{H}_o e^{j\omega t}$

After differentiating $\bar{E}$ & $\bar{H}$ with respect to time & putting the results in equations (1) & (2) we have

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$$\bar{\nabla}\times\bar{H} = j\omega\epsilon\bar{E} \quad \text{——(3)}$$

$$\bar{\nabla}\times\bar{E} = -j\omega\mu\bar{H} \quad \text{——(4)}$$

EM-Wave equations for non-conducting media we have

$$\nabla^2\bar{E} = -\omega^2\mu\epsilon\bar{E} \quad \text{——(5)}$$

$$\nabla^2\bar{H} = -\omega^2\mu\epsilon\bar{H} \quad \text{——(6)}$$

Refer to previously done topic EM-Wave equation for sinusoidal variations.

Let us express now equations (3) & (4) in to its x, y & z components. We have

From equation (3) & (4)

$$\left(\cancel{\frac{\partial H_z}{\partial y}} - \frac{\partial H_y}{\partial z}\right) = j\omega\epsilon E_x \quad \text{——(7)}$$

$$\left(\cancel{\frac{\partial E_z}{\partial y}} - \frac{\partial E_y}{\partial z}\right) = -j\omega\mu H_x \quad \text{——(8)}$$

$$\left(\frac{\partial H_x}{\partial z} - \cancel{\frac{\partial H_z}{\partial x}}\right) = j\omega\epsilon E_y \quad \text{——(9)}$$

$$\left(\cancel{\frac{\partial E_x}{\partial z}} - \frac{\partial E_z}{\partial x}\right) = j\omega\mu H_y \quad \text{——(10)}$$

$$\left(\frac{\partial H_y}{\partial x} - \cancel{\frac{\partial H_x}{\partial y}}\right) = j\omega\epsilon E_z \quad \text{——(11)}$$

$$\left(\frac{\partial E_y}{\partial x} - \cancel{\frac{\partial E_x}{\partial y}}\right) = -j\omega\mu H_z \quad \text{——(12)}$$

* The cancellations of above terms are explained in the next page..

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Similarly from eq (5) & (6)
we have

$$\frac{\partial^2 \bar{E}}{\partial x^2} + \frac{\partial^2 \bar{E}}{\partial y^2} + \frac{\partial^2 \bar{E}}{\partial z^2} = -\omega^2 \mu \epsilon \bar{E} \quad (13)$$

$$\frac{\partial^2 \bar{H}}{\partial x^2} + \frac{\partial^2 \bar{H}}{\partial y^2} + \frac{\partial^2 \bar{H}}{\partial z^2} = -\omega^2 \epsilon \mu \bar{H} \quad (14)$$

* In equations from (7 – 12) those terms of $\bar{E}$ & $\bar{H}$ vanishes which extend along y where $\bar{E}$ & $\bar{H}$ fields are considered to be constant as being out side of the bounded region.

Assuming that wave is propagating in the +ve z-direction as shown in fig-1 and the variation of all field components in this direction may be expressed in the form of $e^{-\gamma z}$ where $(\gamma = \alpha + j\beta)$ — (14-A) is a complex wave propagation.

When we include the time variation part as $e^{-\gamma z} \cdot e^{j\omega t} = e^{(j\omega t - \gamma z)}$ represents the space-time propagation of EM-wave in the z-direction

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Since the Space along y-direction extends to inifinite direction where no boundary conditions to be met we can assume that the fields in the y-direction are uniform and hence all partial derivatives with respect to "y" vanishes ie in equations (7 to 12).

Let $E_y = E_{y_0} e^{-z\gamma}$

in general $\bar{E} = E_0 e^{-\gamma z}$ & $\bar{H} = H_0 e^{-\gamma z}$

also we can have the partial differential of 'E_y' with respect to 'z'

as $\dfrac{\partial E_y}{\partial z} = -\gamma E_0 e^{-\gamma z} = -\gamma E_y$ — (15)

Similarly we can write the corresponding results for z-derivatives of other field components. Making use of these results & fact that all derivatives of 'y' out side the bounded region are zero we have the equations from equations (7-12) and (13-14)

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$$\gamma H_y = j\omega\epsilon E_x \quad \text{——(16)} \quad \text{Ref: From eq(7)}$$

$$\gamma E_y = -j\omega\mu H_x \quad \text{——(17)} \quad \text{" " (8)}$$

$$-\gamma H_x = -\frac{\partial H_z}{\partial x} = j\omega\epsilon E_y \quad \text{—(18)} \quad \text{" " (9)}$$

$$-\gamma E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \quad \text{—(19)} \quad \text{" " (10)}$$

$$\frac{\partial H_y}{\partial x} = j\omega\epsilon E_z \quad \text{——(20)} \quad \text{" " (11)}$$

$$\frac{\partial E_y}{\partial x} = -j\omega\mu H_z \quad \text{——(21)} \quad \text{" " (12)}$$

$$\frac{\partial^2 \bar{E}}{\partial x^2} + \gamma^2 \bar{E} = -\omega^2 \epsilon\mu \bar{E} \quad \text{—(22)} \quad \text{" " (13)}$$

$$\frac{\partial^2 \bar{H}}{\partial x^2} + \gamma^2 \bar{H} = -\omega^2 \epsilon\mu \bar{H} \quad \text{—(23)} \quad \text{--- (14)}$$

Solving the equations (16–21)

Let us solve for 'H_x'

from eq (18) we have

$$-\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y$$

Re-arrange the above & putting

$$E_y = -\frac{j\omega\mu H_x}{\gamma} \quad \text{from eq(17)}$$

$$-\frac{\partial H_z}{\partial x} = \gamma H_x + j\omega\epsilon \left[-\frac{j\omega\mu H_x}{\gamma}\right]$$

we have

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$$-\frac{\partial H_z}{\partial x} = \frac{H_x}{\gamma}\left(\gamma^2 + \omega^2 \epsilon \mu\right)$$

$$-\frac{\partial H_z}{\partial x} = \frac{H_x}{\gamma} l^2 \quad \text{where} \left[l^2 = \gamma^2 + \omega^2 \epsilon \mu \right]$$

$$H_x = \left[-\frac{\gamma}{l^2} \frac{\partial H_z}{\partial x} \right] \quad \text{——} (24)$$

Similarly we can get

$$E_x = -\frac{\gamma}{l^2} \frac{\partial E_z}{\partial x} \quad \text{——} (25)$$

$$H_y = -\frac{j\omega\epsilon}{l^2} \frac{\partial E_z}{\partial x} \quad \text{——} (26)$$

&

$$E_y = \frac{j\omega\mu}{l^2} \frac{\partial H_z}{\partial x} \quad \text{——} (27)$$

where $l^2 = \gamma^2 + \omega^2 \epsilon \mu$ ——(28)

Hence the equations (24–27) represents the plane wave equations in its components while propagating along +ve z-direction.

It can be seen from above equations (24–27) that there must be a z-component of $\bar{E}$ & $\bar{H}$ otherwise all the components will be zero & there will be no fields in space.