(parallel Conducting planes)

TE & TM Modes.

It is convenient to divide the Solution of (EM-waves) in two sets i.e in TE & TM i.e Transe Electric (TE-Mode) & Trans Magnetic (TM–Mode).

(i) Trans Electric Mode: In Trans Electric mode the Electric field is transverse to the direction of propagation with no $\vec{E}$ longitudinal component $E_z = 0$ & $H_z \neq 0$ with having components E_x, E_y & $H_z \neq 0$

(ii) Trans Magnetic Mode: In Trans Magnetic mode the magnetic field is transverse to the direction of propagation with no longitudinal Components of field $\vec{H}$ i.e $H_z = 0$ but $E_z \neq 0$ with Components H_x, H_y & E_z.

TE – Mode: ($E_z = 0$). $H_z \neq 0$

We have equations (22 & 23)

$$\frac{\partial^2 \bar{E}}{\partial x^2} + \gamma^2 \bar{E} = -\omega^2 \epsilon \mu \bar{E} \quad -22$$

&

$$\frac{\partial^2 \bar{H}}{\partial x^2} + \gamma^2 \bar{H} = -\omega^2 \epsilon \mu \bar{H} \quad -(23)$$

Since $E_y = \bar{E}_{y_0} e^{-\gamma z}$ & expression (22) in terms of E_y we have

$$\frac{\partial^2 E_y}{\partial x^2} + \gamma^2 E_y = -\omega^2 \epsilon \mu E_y \quad -(29)$$

$$\frac{\partial^2 E_{y_0}}{\partial x^2} = -E_{y_0} (\gamma^2 + \omega^2 \epsilon \mu) = -E_{y_0} l^2 \quad -(30)$$

$$\left(\frac{\partial^2 E_{y_0}}{\partial x^2} = -E_{y_0} l^2 \right) \quad - 31$$

The solution of the second order differential (SHM) equat (31) is given as

$$E_{y_0} = (C_1 \sin(lx) + C_2 \cos lx) \quad -(32)$$

Since $E_y = E_{y_0} e^{-\gamma z}$

$$E_y = (C_1 \sin(lx) + C_2 \cos(lx)) e^{-\gamma z} \quad -(33)$$

The constants C_1 & C_2 can be determined

Contnd.

from the boundary conditions

For parallel plane wave guide the boundary conditions are

at (i) $x=0$ $E_y=0$ as $0<x<a$

(ii) $x=a$ $E_y=0$

Applying the boundary conditions to eq (33),

at $x=0$ $E_y=0$ gives $C_2=0$

at $x=a$ $E_y=0$ gives $l=\left(\frac{m\pi}{a}\right)$

where $m=1,2,3\cdots$

or $la=m\pi$

So we get the solution for E_y

$$E_y = C_1 \sin\left(\frac{m\pi x}{a}\right) e^{-\gamma z} \quad (34)$$

putting "E_y" in eq-(21) $\rightarrow \frac{\partial E_y}{\partial x} = -j\omega\mu H_z$

$$\frac{\partial E_y}{\partial x} = +\frac{m\pi}{a} C_1 \cos\left(\frac{m\pi x}{a}\right) e^{-\gamma z} = -j\omega\mu H_z$$

$$H_z = -\frac{m\pi}{j\omega\mu a} C_1 \cos\left(\frac{m\pi x}{a}\right) e^{-\gamma z} \quad (35)$$

& putting "E_y" in equation (17) $\rightarrow \gamma E_y = -j\omega\mu H_x$

we have $H_x = -\frac{\gamma}{j\omega\mu} C_1 \sin\left(\frac{m\pi x}{a}\right) e^{-\gamma z} \quad (36)$

Contd.

The value of "m" specifies particular field configuration or mode

Referring to $\gamma = \alpha + j\beta$, we know that the smallest value for $m=1$ as if "m" become zero $m=0$ makes all fields identically zero. Thus the lowest order mode that can exists in this case is TE_1. further we know that waves are guided by perfectly conducting walls, "γ" is either real i.e α has a value & β is zero, so that there is an attenuation but no phase shift and therefore no wave propagation. However when "γ" is imaginary α is zero & β has a value which corresponds to the propagation of wave without attenuation.

So for propagation $\gamma = j\beta$ as $\rightarrow \alpha = 0$

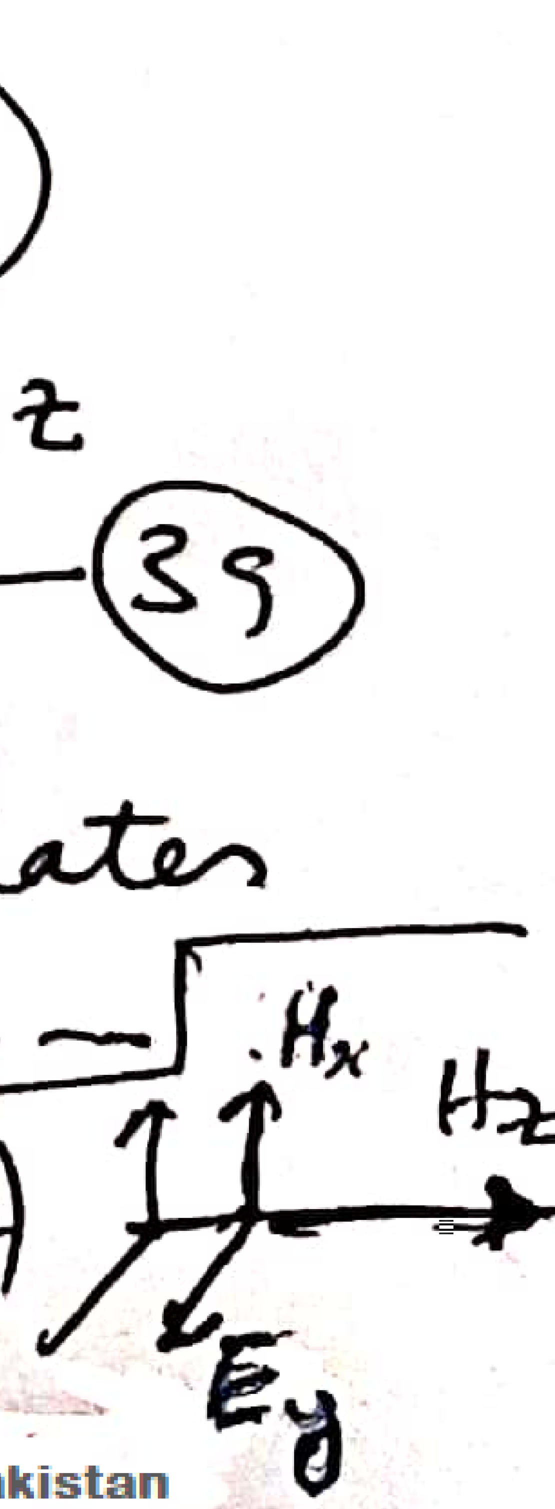
$$E_y = C_1 \sin\left(\frac{\pi m}{a} x\right) e^{-j\beta z} \quad (37)$$

$$H_x = -C_1 \frac{\beta}{\mu\omega} \sin\left(\frac{\pi m}{a} x\right) e^{-j\beta z} \quad (38)$$

$$H_z = \frac{j m \pi}{\omega \mu a} C_1 \cos\left(\frac{m\pi x}{a}\right) e^{-j\beta z} \quad (39)$$

In the above equations "a" is the plates separation between the two & $m = 1, 2, 3 \ldots$ & m and a are related by

(TE)



$$\beta = \frac{m\pi}{a} \quad (40)$$