

EMWG-52

Attenuation in Wave guide: " γ_m "
(parallel conducting planes)

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The attenuation or propagation wave constant γ_m is given as

$$\gamma_m = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \epsilon \mu} \quad \text{--- (1)}$$

Also

($\gamma_m = \alpha_m + j\beta_m$) depends on the values under the radical & can be wholly real or imaginary depending on the relative magnitudes of $\left(\frac{m\pi}{a}\right)$ & $(\omega^2 \epsilon \mu)$.

Thus two conditions arise with α_m equal to zero or β_m equal to zero. It is possible to have propagation with zero attenuation $\alpha = 0$ or high attenuation with $\beta = 0$.

It is also seen that frequency of the wave is also another major criteria determining whether the wave propagation is taking place or not.

Contd:

The critical frequency
at which the wave starts travelling
along the wave guide or that the
attenuation condition changes to that
of propagation can be determined
from equation-(1)

let us find the condition for " f_c " the
critical frequency when $\gamma_m = 0$

$$\sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega_c^2 \epsilon \mu} = 0 \quad \text{--- (2)}$$

let us find " f_c " from above eqn

$$\omega_c = 2\pi f_c$$

$$\left(\frac{m\pi}{a}\right)^2 - (2\pi f_c)^2 \epsilon \mu = 0 \quad \left| \begin{array}{l} \omega_c \sqrt{\epsilon \mu} = \left(\frac{m\pi}{a}\right) \end{array} \right.$$

$$f_c = \frac{m}{2a} \sqrt{\frac{1}{\epsilon \mu}}$$

$$\text{Also we have } \epsilon \mu = \frac{m^2}{(2a f_c)^2} \text{ put in (1)}$$

$$\gamma_m = \sqrt{\left(\frac{m\pi}{a}\right)^2 - (2\pi f)^2 \epsilon \mu}$$

$$= \sqrt{\left(\frac{m\pi}{a}\right)^2 - (2\pi f)^2 \times \frac{m^2}{(2a f_c)^2}}$$

$$= \left(\frac{m\pi}{a}\right) \sqrt{1 - \left(\frac{f}{f_c}\right)^2} \quad \text{--- (3)}$$

$$\text{Also } \left(\frac{m\pi}{a}\right) = \omega_c \sqrt{\epsilon \mu}$$

$$\gamma_m = \omega_c \sqrt{\epsilon \mu} \sqrt{1 - \left(\frac{f}{f_c}\right)^2} \quad (4) \quad 3/5$$

for frequency below cutoff where $f < f_c$, γ_m is real the fields are attenuated & $\gamma_m = \alpha_m$ $\beta_m = 0$.

The field amplitude decreases very rapidly with distance 'z' because attenuation term $e^{-\gamma_m z}$ even though the phase will be constant.

For frequencies above cutoff $f > f_c$ γ_m is imaginary & $\alpha_m = 0$ while $\gamma_m = j\beta_m$

$$\begin{aligned} \gamma_m &= j \left(\frac{m\pi}{a} \right) \sqrt{\left(\frac{f}{f_c}\right)^2 - 1} \\ &= j \omega_c \sqrt{\epsilon \mu} \sqrt{\left(\frac{f}{f_c}\right)^2 - 1} \quad (5) \end{aligned}$$

This condition is of interest because wave propagation occurs without attenuation in the z-direction.

In practical situations owing to the finite conductivity of plates

Continued

Some attenuation accompanies
at frequency above cut off frequencies.

A physical significance can be
attributed to the critical frequency
of cut off by writing expression

$$f_c = \frac{m}{2a} \sqrt{\frac{1}{\epsilon \mu}} \quad \text{as } \frac{1}{\sqrt{\epsilon \mu}} = v$$

$$f_c = \frac{m}{2a} v \quad \text{--- (6)}$$

$$\text{as } v = f_c \lambda_c$$

$$\& \lambda_c = v / f_c$$

$$\lambda_c = \frac{2a}{m}$$

$$\text{or } a = \frac{m \lambda_c}{2} \quad \text{--- (7)}$$

The equatio (7), gives the relation between
the plate separation "a" and the critical
wave length " λ_c ". Thus the critical
wave length is fixed by the plate's spacing
as that at which the distance between
the plates becomes exactly "m" half
of the wave length.

Contd:

For lower frequency or long wave length, are attenuated and at higher frequencies or shorter wave length, are propagated with out loss.

The largest value of the cut off wave length is $2a$ when $m=1$

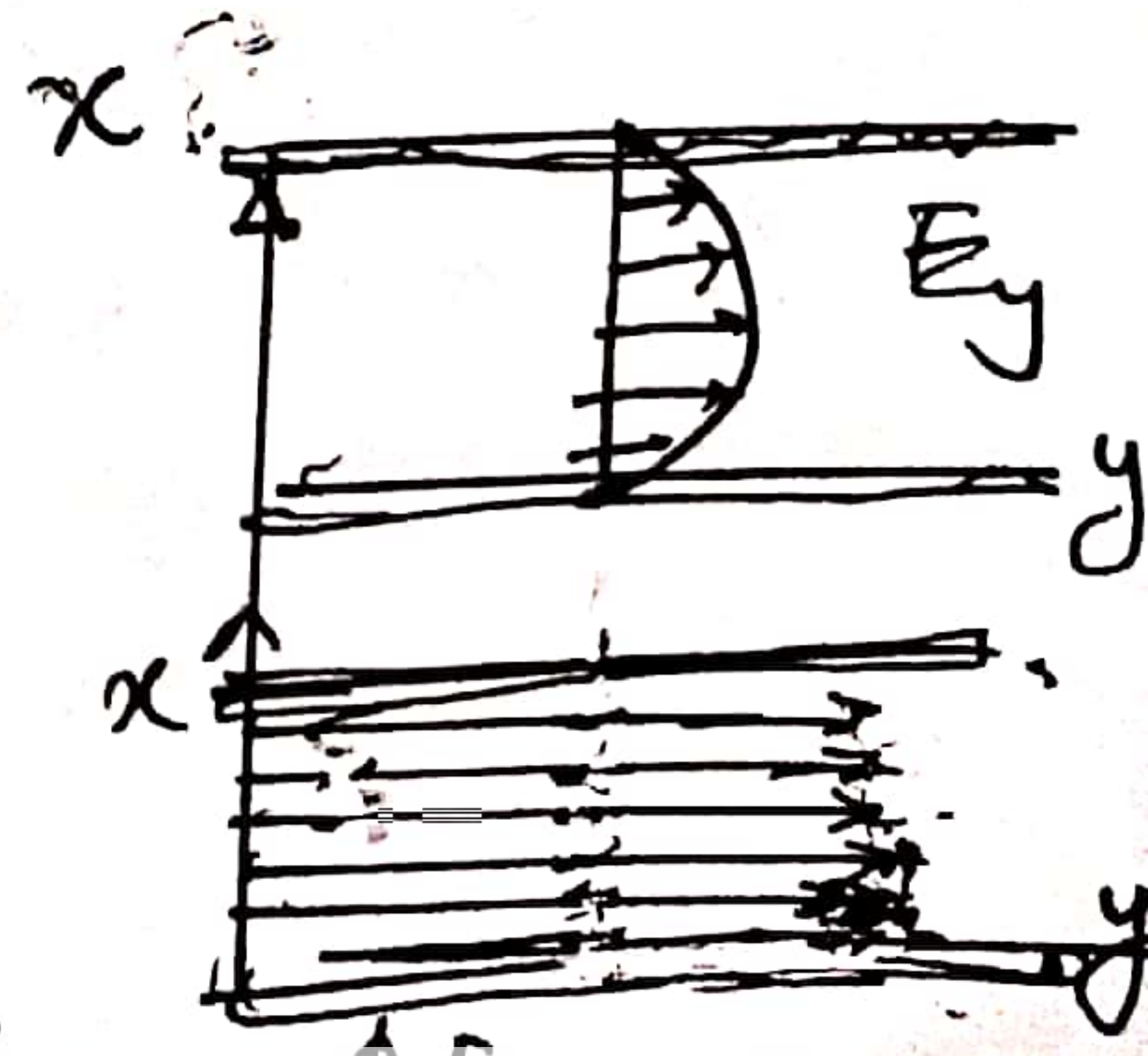
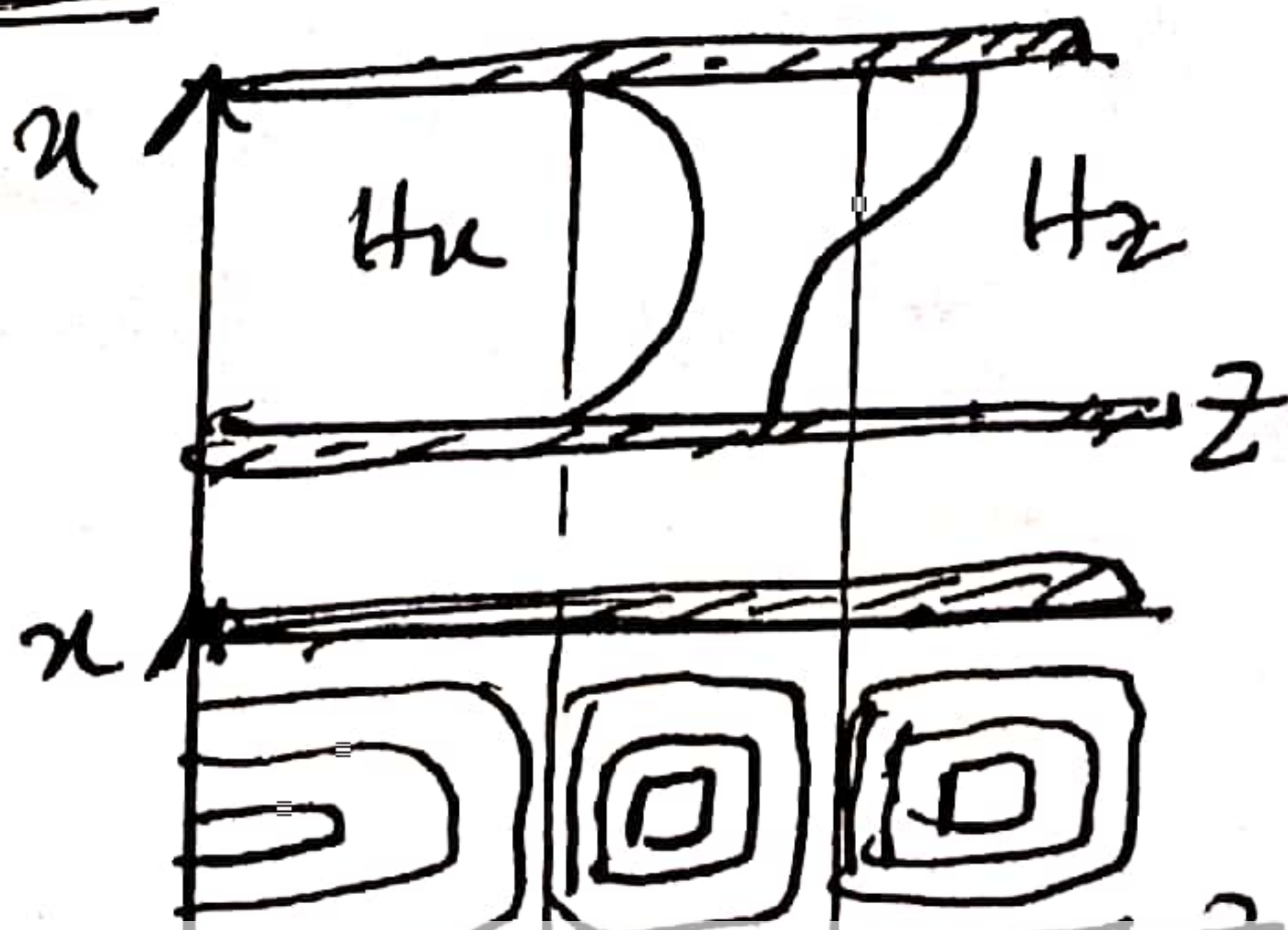
This means that the longest free space wave length a signal may have & still be propagating in parallel plane guide is just less than plane separation.

The field component of TE_m wave with propagation purely imaginary part

$$\text{given } E_y = C_1 \sin\left(\frac{m\pi}{a}x\right) e^{-j\beta_m z} \quad \text{--- (8)}$$

$$H_x = -\frac{\beta_m C_1 \sin\left(\frac{m\pi}{a}x\right)}{\omega\mu} e^{-j\beta_m z} \quad \text{--- (9)}$$

Mode $(TE_1)_0$ $\rightarrow H_z = \frac{j m \pi}{a \omega \mu} C_1 \cos\left(\frac{m\pi}{a}x\right) e^{-j\beta_m z} \quad \text{--- (10)}$



EM fields between parallel plates