EMWGS3

## Phase velocity and Group Velocity: ($V_p$, $V_g$)

The phase velocity of wave progressing with constant phase.
Let us take an EM-wave travelling in the z-direction has constant phase $(-\beta z + \omega t)$

$$-\beta z + \omega t = \text{Const}: \quad \text{---} (1)$$

differentiating equation (1) with respect to time

$$\frac{d}{dt}(-\beta z + \omega t) = 0$$

$$\beta \frac{dz}{dt} - \omega = 0$$

$$v_p \beta = \omega$$

Phase velocity is $$v_p = \frac{\omega}{\beta} \quad \text{---} (2)$$

When a signal is conveyed from one point to another it is always necessary to modulate (Mix) by some other means of a

Contd.

Carrier frequency being transmitted.

After modulation we have a group of frequencies centred around carrier signal that must be transmitted along the guide.

If a phase velocity is a function of frequency, different frequencies in a group will have different velocities. The wave combined to form a modulation envelope which is propagating as a wave having a group velocity $V_g$ defined as

$$V_g = \frac{d\omega}{d\beta} \quad -(2)$$

we have $\quad \gamma^2 = l^2 - \omega^2 \epsilon \mu \quad -(3)$

Since $\gamma = j\beta$, $\gamma^2 = -\beta^2$

$$-\beta^2 = l^2 - \omega^2 \epsilon \mu$$

$$\beta^2 = \omega^2 \epsilon \mu - l^2 \quad \text{But } l = \frac{m\pi}{a}$$

$$\beta^2 = \omega^2 \epsilon \mu - \frac{m\pi}{a} \quad -(4)$$

Differentiating eq(4) w.r.t. "$\omega$"

$$2\beta \frac{d\beta}{d\omega} = 2\omega \epsilon \mu - 0$$

$$\frac{d\beta}{d\omega} \beta = \omega \epsilon \mu \quad -(5)$$

(Contd.)

Since $v_p = \frac{\omega}{\beta}$

& $\epsilon\mu = \frac{1}{v^2}$

putting in equation (5)

$$\beta \frac{1}{v_g} = \omega \epsilon \mu$$

$$\frac{1}{v_g} = \frac{\omega}{\beta}\left(\frac{1}{v^2}\right)$$

$$v^2 = v_p v_g$$

where "v" is the speed of the wave with combined velocities $v_p$ & $v_g$.