Rectangular Wave Guide:

A rectangular wave guide is formed by placing two more conducting planes on the open sides of a two parallel conducting planes. Such rectangular tube formed, is called a rectangular wave guide. as shown in Fig-1

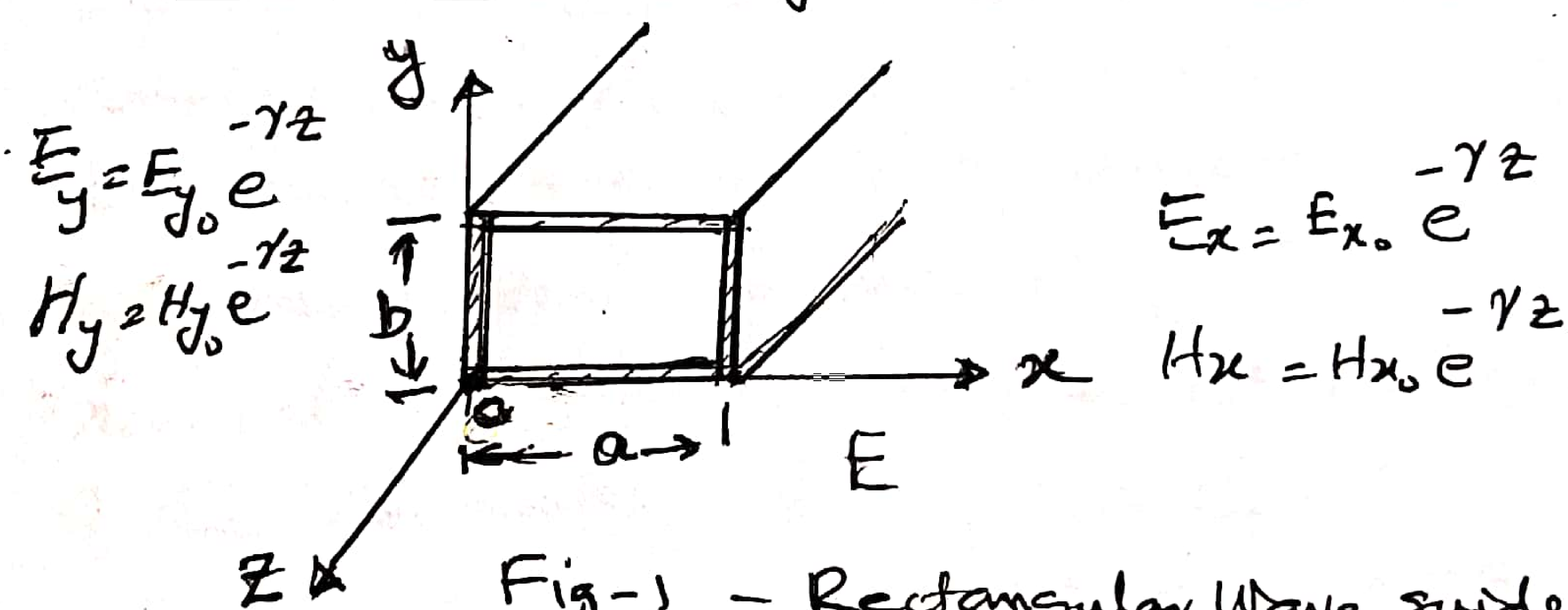

Fig-1 - Rectangular Wave guide.

The rectangular wave guide and all such similar ones, provide an alternative transmission lines at ultrahigh frequencies for the transmission of electrical energy.

Any wave propagating with in such a wave guide must satisfy the Maxwell's equations and must also satisfy the boundary

Contd.

Conditions imposed by the walls of wave guide.

Let us consider "a" & "b" be the plate separations between two parallel conducting planes along "x" and "y" axes respectively.

To determine the field configuration in the guide, Maxwell's equations are solved subject to the appropriate boundary conditions. Assuming that the walls of the guide are made of perfect conductors, the tangential components of $\bar{E}$ and normal components are zero at the surface of the conductors. ie

$$\begin{cases} H_n = E_t = 0 \\ \text{at } x=0 \quad x=a \\ \text{at } y=0 \quad y=b \end{cases}$$

The Maxwell's equations for Electromagnetic wave travelling in the wave guide filled with air, being a non-conducting medium are given as

$$\bar{\nabla} \times \bar{H} = \epsilon \frac{\partial \bar{E}}{\partial t} \quad \text{——— (1)}$$

$$\bar{\nabla} \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t} \quad \text{——— (2)}$$

Contd:

let $\bar{E} = \bar{E}_o e^{j\omega t}$

& $\bar{H} = \bar{H}_o e^{j\omega t}$

$\frac{\partial \bar{E}}{\partial t} = j\omega\epsilon E$, $\frac{\partial \bar{H}}{\partial t} = j\omega\mu\bar{H}$

Putting the above terms in equation (1) & (2)

$\bar{\nabla}\times\bar{H} = j\omega\epsilon\bar{E}$ —— (3)

$\bar{\nabla}\times\bar{E} = -j\omega\mu\bar{H}$ —— (4)

taking curl of both sides of equation (3) & (4) and applying vector ID

$\bar{\nabla}\times\bar{\nabla}\times\bar{A} = \bar{\nabla}(\bar{\nabla}\cdot\bar{A}) - \nabla^2\bar{A}$ we get

$\nabla^2\bar{H} = -\omega^2\epsilon\mu\bar{H}$ —— (5) $\because \bar{\nabla}\cdot\bar{H} = 0$

& $\nabla^2\bar{E} = -\omega^2\epsilon\mu\bar{E}$ —— (6) $\because \bar{\nabla}\cdot\bar{E} = 0$ $(\rho = 0)$

Expressing equations (3) & (4) in Components

$\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial H_z}\right) = j\omega\epsilon E_x$ — (7)

$\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) = j\omega\epsilon E_z$ — (8)

$\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) = j\omega\epsilon E_y$ — (9)

Contd:

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x \quad (10)$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \quad (11)$$

$$\frac{\partial E_y}{\partial} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \quad (12)$$

& similarly for equation (5) & (6)

$$\frac{\partial^2 \bar{E}}{\partial x^2} + \frac{\partial^2 \bar{E}}{\partial y^2} + \frac{\partial^2 \bar{E}}{\partial z^2} = -\omega^2 \epsilon\mu \bar{E} \quad (13)$$

$$\& \quad \frac{\partial^2 \bar{H}}{\partial x^2} + \frac{\partial^2 \bar{H}}{\partial y^2} + \frac{\partial^2 \bar{H}}{\partial z^2} = -\omega^2 \epsilon\mu \bar{H} \quad (14)$$

let $\gamma = \alpha + j\beta$

and the EM-Wave travel as $e^{-\gamma z}$ with $e^{j\omega t}$

Space-time dependent parts

$$\bar{E} = \bar{E}_o \, e^{(-\gamma z + j\omega t)}$$

$$\& \quad \bar{H} = \bar{H}_o \, e^{(-\gamma z + j\omega t)}$$

Now we have $\bar{E}$ & $\bar{H}$ plane wave components in the wave guide

as $E_y = E_{oy} e^{-\gamma z}$, $H_x = H_{ox} e^{-\gamma z}$

$E_x = E_{ox} e^{-\gamma}$ $\quad H_y = H_{oy} e^{-\gamma z}$

The space dependent parts of the EM-Wave

Contd:

putting $\frac{\partial H_y}{\partial z} = -\gamma H_y$ in eq (7)

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\epsilon E_x$$

$$\frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\epsilon E_x \quad \text{——} (15)$$

From equation (8)

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z \quad \text{——} (16)$$

From equation (9) we have

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y$$

Since $H_x = H_{0x} e^{-\gamma z}$ $\frac{\partial H_x}{\partial z} = -\gamma \bar{H}_{\cancel{x}}$ put above

$$\frac{\partial H_z}{\partial x} + \gamma H_x = -j\omega\epsilon E_y \quad \text{——} (17)$$

From equation (10) putting

$\frac{\partial E_y}{\partial z} = -\gamma E_y$, $\frac{\partial E_x}{\partial z} = -\gamma E_x$ we have

$$\frac{\partial E_z}{\partial y} - \gamma E_y = -j\omega\mu H_x \quad \text{——} (18)$$

Similarly in equation (11) putting $\frac{\partial E_x}{\partial z} = -\gamma E_x$

$$\frac{\partial E_z}{\partial x} + \gamma E_x = j\omega\mu H_y \quad \text{——} (19)$$

contd:

From eq (12)

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \quad (20)$$

Equations (13) & (14) expressed for the EM-wave travelling in the z-direction

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z = -\omega^2 \epsilon\mu E_z \quad (21)$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z = -\omega^2 \epsilon\mu H_z \quad (22)$$

The above set of equations can be re-arranged to get " H_x " =

From eq. (17)

$$\frac{\partial H_z}{\partial x} = -\gamma H_x - j\omega\epsilon E_y \quad (23)$$

But from eqn (18)

$$E_y = -\frac{1}{\gamma}\frac{\partial E_z}{\partial y} - \frac{j\omega\mu H_x}{\gamma} \quad \text{put in (23)}$$

we get

$$\frac{\partial H_z}{\partial x} = -\frac{H_x}{\gamma}\left[\gamma^2 + \omega^2\epsilon\mu\right] + \frac{j\omega\epsilon}{\gamma}\frac{\partial E_z}{\partial y}$$

Re-arranging and simplifying

$$H_x = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial y} \quad (24)$$

Where $h^2 = \gamma^2 + \omega^2\epsilon\mu$

Contd:

Similarly we can find

$$H_y = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial x} \quad \text{---(25)}$$

$$E_x = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} \quad \text{---(26)}$$

$$E_y = \frac{\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} \quad \text{---(27)}$$

where $h^2 = \gamma^2 + \omega^2 \epsilon\mu$

It can be seen from equation (2 -27) that if E_z & H_z are zero then all field components vanish. Thus for propagation along a wave guide there should be at least one of the E_z or H_z components, present.

The boundary conditions to be satisfied for rectangular wave guide are

$$\left.\begin{aligned} E_y &= E_z = 0 \quad \text{at } x=0 \text{ \& } x=a \\ E_x &= E_z = 0 \quad \text{at } y=0 \text{ \& } y=b \end{aligned}\right] \text{---(28)}$$

The equations (24-27) describes the $\bar{E}$ & $\bar{H}$ fields configurations inside the wave guide.