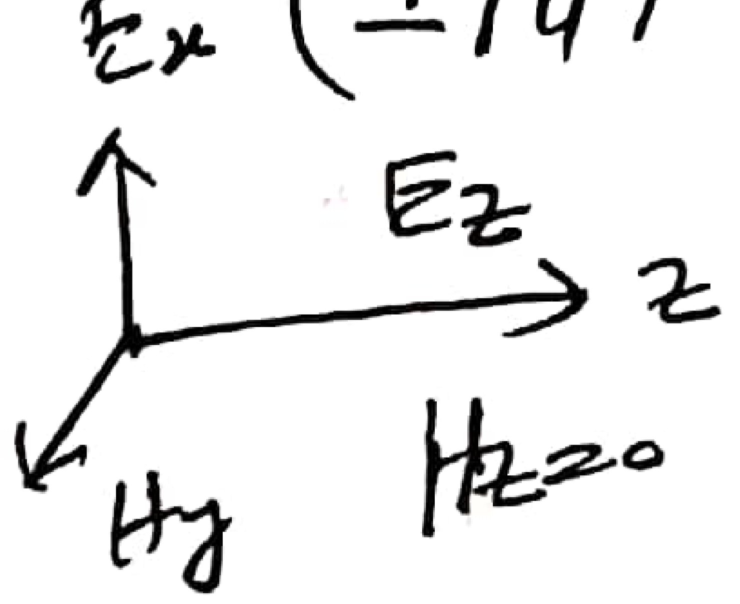
(Rectangular Wave guide):

Transverse Magnentic Waves:



(TM – Mode):

The Wave equation in a wave guide are seen to be partial differential equations which can be solved by assuming product Solution.

We have from equation ——— (6, the E field. as & Equation (21) & is given as

$$\left(\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z = -\omega^2 \epsilon \mu E_z\right)$$

let $\quad E_z = X_{(x)} Y_{(y)}$ ——— (29)

$$\frac{\partial^2 E_z}{\partial x^2} = Y \frac{d^2 X}{dx^2} \quad \& \quad \frac{\partial^2 E_z}{\partial y^2} = X \frac{d^2 Y}{dy^2}$$

From eqn (2) by putting the respective terms we get

$$Y \frac{d^2 X}{dx^2} + X \frac{d^2 Y}{dy^2} + \gamma^2 XY = -\omega^2 \mu \epsilon XY \quad \text{—— (30)}$$

Substituting $h^2 = \gamma^2 + \epsilon \mu \omega^2$ & dividing whole equation by "XY" both sides

$$\frac{1}{X} \frac{d^2 X}{dx^2} + h^2 = -\frac{1}{Y} \frac{d^2 Y}{dy^2} \quad \text{—— (31)}$$

Contd.

Equation (31) Can be Solved by splitting it in to two parts & equating to a Common Constant "A"

$$\frac{1}{X}\frac{d^2X}{dx^2} = -\frac{1}{Y}\frac{d^2Y}{dy^2} = A^2 \quad \text{——— (31a)}$$

$$\frac{1}{X}\frac{d^2X}{dx^2} + h^2 = A^2 \quad \text{——— (32)}$$

$$\frac{1}{Y}\frac{d^2Y}{dy^2} = -A^2 \quad \text{——— (33)}$$

Let $B^2 = h^2 - A^2$

The Solution of equat (32) is of the form given as

$$X = C_1 \cos Bx + C_2 \sin Bx$$

Where $B = \sqrt{h^2 - A^2}$ ——— (33a)

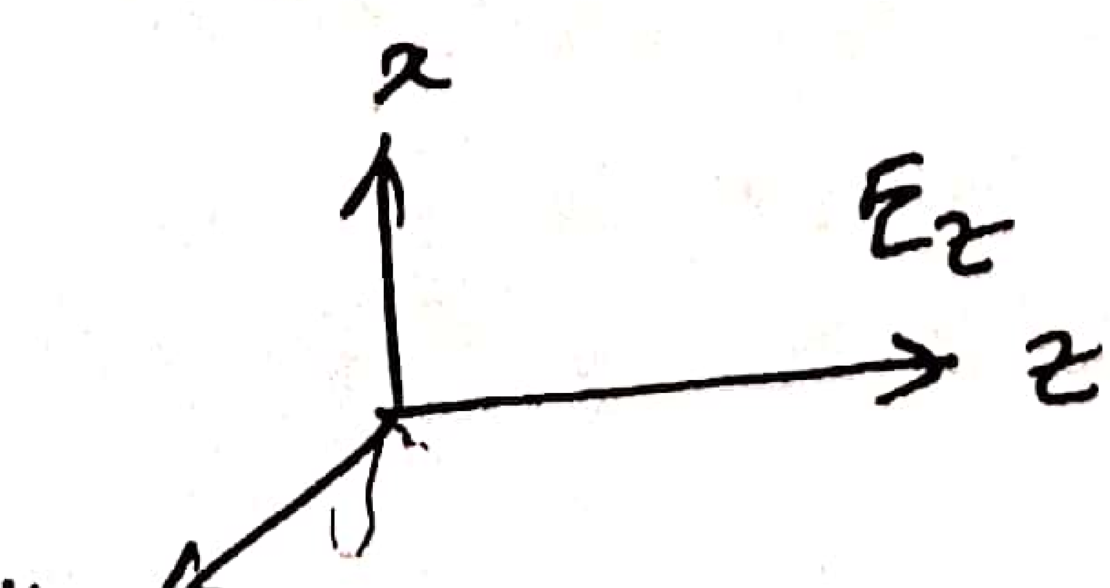
Similarly Solution of equat (33) is of the form

$$Y = C_3 \cos Ay + C_4 \sin Ay \quad \text{——— (34)}$$

When $A = \sqrt{h^2 - B^2}$ ——— (34a)

Now Solving $E_z = XY$

$$E_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay) \quad \text{——— (35)}$$



Contd

$$E_z = C_3 C_4 \cos Bx \cos Ay + C_1 C_4 \cos Bx \sin Ay$$
$$+ C_2 C_3 \sin Bx \cos Ay + C_2 C_4 \sin Bx \sin Ay \quad (36)$$

Applying the Boundary Condition.

at $x = 0$ & for E_z to be zero for all values of y, C_1 should be zero with these conditions imposed the expression for E_z reduces to

$$E_z = C_2 C_3 \sin Bx \cos Ay + C_2 C_4 \sin Bx \sin Ay \quad (37)$$

at $y = 0$

$$E_z = C_2 C_3 \sin Bx \quad (38)$$

Since B can not be zero in the above expression there is a chance that either C_2 or C_3 be zero.

If C_2 is zero the E_z reduces to zero and hence only possibility is that $C_3 = 0$

$$C_3 = 0 \quad (39)$$

we get

$$E_z = C_2 C_4 \sin Bx \sin Ay$$

Since $C_2 C_4$ being constants and may be substituted by a new constant "C"

$$C_1 C_2 = C$$

$$E_z = C \sin Bx \sin Ay \quad (40)$$

(Contd:)

Applying boundary conditions

at $x = a$

$$E_z = C \sin Ba \sin By \quad \text{—— (41)}$$

For E_z to vanish for all values of y, Ba must be integral multiple of π

$$Ba = m\pi \quad \text{where } m = 1, 2, 3 \ldots$$

$$B = \frac{m\pi}{a}$$

at $y = b$

$$E_z = C \sin \frac{m\pi}{a} \sin Ab \quad \text{—— (42)}$$

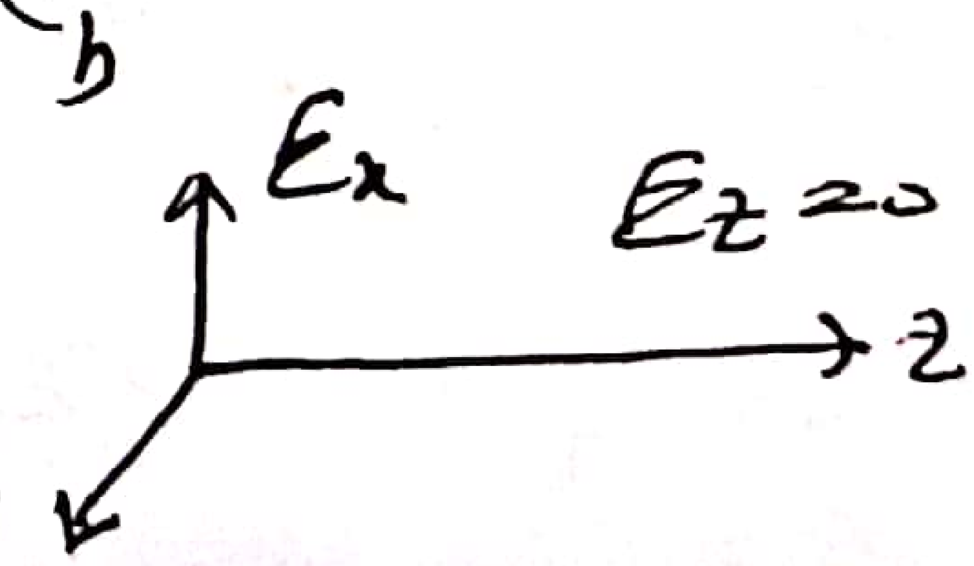
The equat (42) to vanish for all values of $Ab = n\pi$ when $n = 1, 2, 3, \ldots$

or $A = \frac{n\pi}{b}$

Hence the solution for E_z is obtained as

$$E_z = C \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \quad \text{—— (43)}$$

Assignment for Students



For E_x solution in equation 26 putting $H_z = 0$

$E_x = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x}$ can get the solution.

For H_y from eq (25)

$H_y = -\frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial x}$ as we put $H_z = 0$

Solution is obtained