

1 Trans-Electric Mode (TE) 1/2

(Rectangular wave guide)

Considering the Magnetic field vector equation given as Ref Eq-

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z = -\omega^2 \epsilon \mu H_z \quad \text{Eq (22)}$$

let $H_z = X Y$ — (48)

putting it in the equation (22) in the similar way done in (TM) mode we get the relation

$$\frac{1}{X} \frac{d^2 X}{dx^2} + h^2 = -\frac{1}{Y} \frac{d^2 Y}{dy^2} = A^2 \quad \text{(49)}$$

Where A^2 is a common constant

We have $\frac{1}{X} \frac{d^2 X}{dx^2} + B^2 = 0$ — (50) when $B^2 = h^2 = A^2$

The solution of equation (50) is given

$$X = C_5 \cos Bx + C_6 \sin Bx \quad \text{(51)}$$

Contd:

Similarly for Y solution

2/2

$$\frac{1}{Y} \frac{d^2 Y}{dy^2} + A^2 = 0$$

$$Y = C_7 \cos Ay + C_8 \sin Ay \quad (52)$$

$$\text{Since } H_z = XY$$

$$H_z = (C_5 \cos Bx + C_6 \sin Bx)(C_7 \cos Ay + C_8 \sin Ay)$$

$$H_z = C_5 C_7 \cos Bx \cos Ay + C_5 C_8 \cos Bx \sin Ay + C_6 C_7 \sin Bx \cos Ay + C_6 C_8 \sin Bx \sin Ay \quad (53)$$

Applying boundary condition

$$\text{at } x=0 \quad C_8 = 0 \quad C_5 \neq 0 \quad C_6 = 0$$

We have

$$H_z = C \cos Bx \cos Ay \quad (54)$$

For Boundary conditn $x=a$ & $y=b$

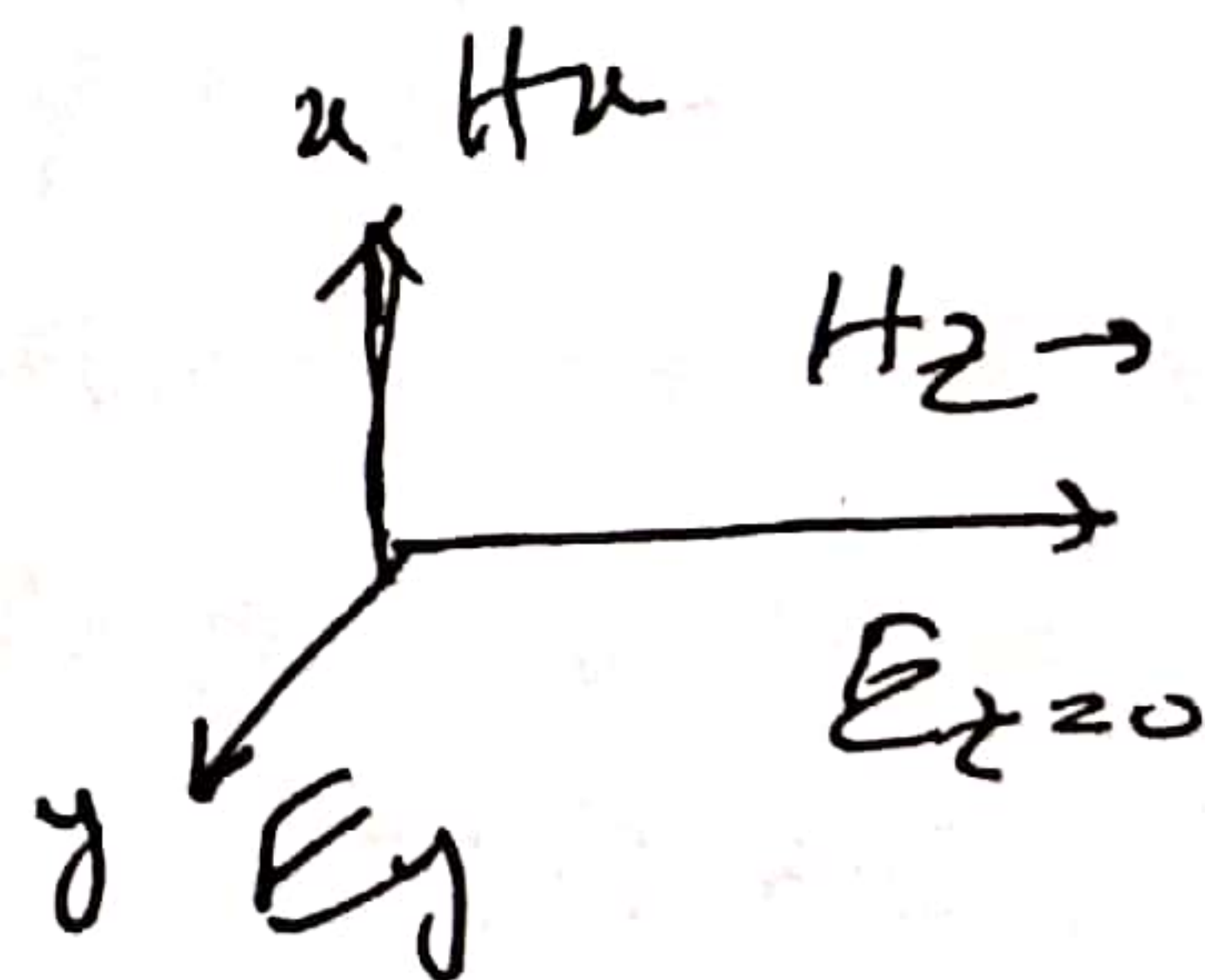
$$H_z = C \cos Ba \cos Ab$$

$$\text{Where } Ba = n\pi$$

$$\text{or } B = \frac{n\pi}{a}$$

$$\Delta \quad Ab = n\pi$$

$$\text{or } A = \frac{n\pi}{a}$$



$$H_z = C \cos\left(\frac{n\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \quad (55)$$

H_x & E_y can be found from Eqs 24 & 27

Assignment for Students