Internal impedance of a Conducting medium :

The intrinsic impedance or characteristic impedance of a non-conducting medium is found to be simply square root of the ratio of parameters of medium namely μ & ϵ.

For free space it is equal to 377Ω

i.e. The impedance Z or $\eta = \frac{E}{H} = \sqrt{\frac{\mu}{\epsilon}}\ (\Omega)$

for free space $\mu = \mu_0$ & $\epsilon = \epsilon_0$ (units)

$Z = \eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\,\Omega$ Also $\frac{E}{H}$ medium $= \frac{V/m}{A/m} = \frac{V}{A} = (\Omega)$

So for free space 377Ω is purely resistive.

If a medium is a conducting one that is having finite conductivity, the impedance of the medium is no more pure resistive but becomes a complex factor.

An expression for the impedance can be derived as follows:

Considering the Maxwell's equations for $\bar{E}$ & $\bar{H}$,

$$\bar{\nabla} \times \bar{E} = -j\omega\mu\bar{H} \quad \text{---(1)}$$

$$\bar{\nabla} \times \bar{H} = (\sigma\bar{E} + j\omega\epsilon\bar{E}) \quad \text{---(2)}$$

Hints

While taking

$\bar{B} = \bar{B}_0 e^{j\omega t}$

$\bar{E} = \bar{E}_0 e^{j\omega t}$ $\quad \bar{B} = \mu\bar{H}, \ \bar{J} = \sigma\bar{E}$

Contd:

For a uniform plane wave propagating along let us say in the x-direction

The yz-plane directions of Curl of $\bar{E}$ & $\bar{H}$ can be expressed as

$$(\bar{\nabla} \times E)_{yz} = -\frac{\partial E_z}{\partial x}\underline{j} + \frac{\partial E_y}{\partial x}\hat{k} \quad -(3)$$

Note we take in the $\bar{\nabla}\times\bar{E}$ the terms of E_y which vary with "x" while all other E terms considered constants & their partial derivatives are considered be zero.

$$(\bar{\nabla} \times H)_{yz} = -\frac{\partial H_z}{\partial x}\underline{j} + \frac{\partial H_y}{\partial x}\hat{k} \quad -(4)$$

Substituting equations (3) & (4) in equations (1) & (2) respectively

$$-\frac{\partial E_z}{\partial x}\underline{j} + \frac{\partial E_y}{\partial x}\hat{k} = j\omega\mu(H_y\hat{j} + H_z\hat{k}) \quad -(5)$$

Where $\bar{E}_{yz} = E_y\hat{j} + E_z\hat{k}$, & $\bar{H}_{yz} = H_y\hat{j} + H_z\hat{k}$

$$-\frac{\partial H_z}{\partial x}\hat{j} + \frac{\partial H_y}{\partial x}\hat{k} = (\sigma + j\omega\epsilon)(E_y\hat{j} + E_z\hat{k}) \quad -(6)$$

Equating the terms in the equations (5) & (6) & re-arranging we get

$$-\frac{\partial H_z}{\partial x} = (\sigma + j\omega\epsilon)E_y \quad -(7)$$

$$\frac{\partial E_z}{\partial x} = j\omega\mu H_y \quad -(8)$$

Contd:

$$\frac{\partial H_y}{\partial x} = (\sigma + j\omega\epsilon) E_z \text{ --- } (9)$$

$$\frac{\partial E_y}{\partial x} = -j\omega\mu H_z \text{ --- } (10)$$

Dividing equation (7) by (10) we have

$$\frac{\partial E_y}{\partial H_z} = \frac{j\omega\mu}{\sigma + j\epsilon\omega}\left(\frac{H_z}{E_y}\right) \text{ --- } (11)$$

Re-arrange eq (11)

$$E_y \, \partial E_y = \frac{j\omega\mu}{\sigma + j\omega\epsilon} H_z \, \partial H_z \text{ --- } (12)$$

Intergrating the both sides of eq (12) & ignoring constant

$$\int E_y \, \partial E_y = \frac{j\omega\mu}{\sigma + j\omega\epsilon} \int H_z \, \partial H_z$$

$$\frac{(E_y)^2}{\cancel{2}} = \frac{j\omega\mu}{\sigma + j\omega\epsilon} \frac{(H_z)^2}{\cancel{2}}$$

$$\left(\frac{E_y}{H_z}\right)^2 = \frac{j\omega\mu}{\sigma + j\omega\epsilon}$$

$$\frac{E_y}{H_z} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \text{ --- } (13)$$

Hence the ratio of $\bar{E}$ & $\bar{H}$ is written as

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$$Z \overset{or}{=} \eta = \frac{E}{H} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \quad \text{——(14)}$$

which is the impedance of the conducting medium with finite conductivity.
for good conductors $\sigma >> \epsilon\omega$ the expression (14) reduces to

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} \quad \text{——(15)}$$

$$\eta = \sqrt{j}\left(\sqrt{\frac{\omega\mu}{\sigma}}\right) \qquad \sqrt{j} = \left(\frac{1+j}{\sqrt{2}}\right)$$

$$\eta = \left(\frac{1+j}{\sqrt{2}}\right)\left[\sqrt{\frac{\omega\mu}{\sigma}}\right]$$

$$\eta = \sqrt{\frac{\omega\mu}{2\sigma}} + j\sqrt{\frac{\omega\mu}{2\sigma}}$$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ \quad \text{——(16)}$$

From the equation (16) it follows that the magnitude of intrinsic impedance of a conductor is given by $\eta = \sqrt{\frac{\omega\mu}{\sigma}}$ with the phase difference of 45°.

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At a frequency of 3 GHz the magnitude of intrinsic impedance for copper is found to be 0.020 Ω ie

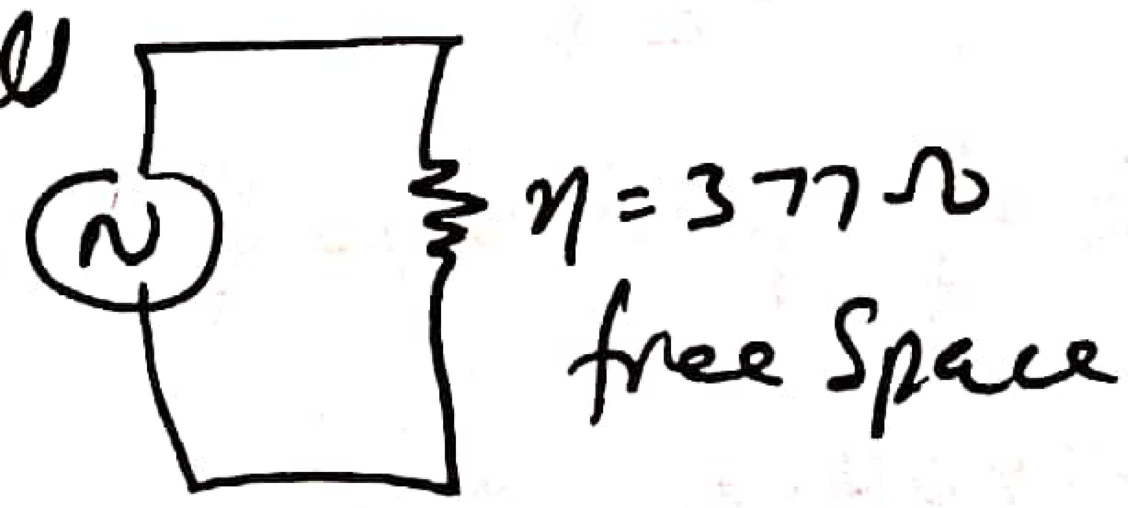
$$|\eta|_{Cu} = 0.020\,\Omega$$

This shows that for a conducting medium such as copper the ratio E to H is much less than for free space.

for a perfect conductor σ is infinite & "η" ie intrinsic impedance tends to zero hence E vanishes $\bar{E} = 0$

The small value of "η" suggests that a conducting medium like copper behaves as a short circuit to EM-fields

For $\bar{E}$ field only



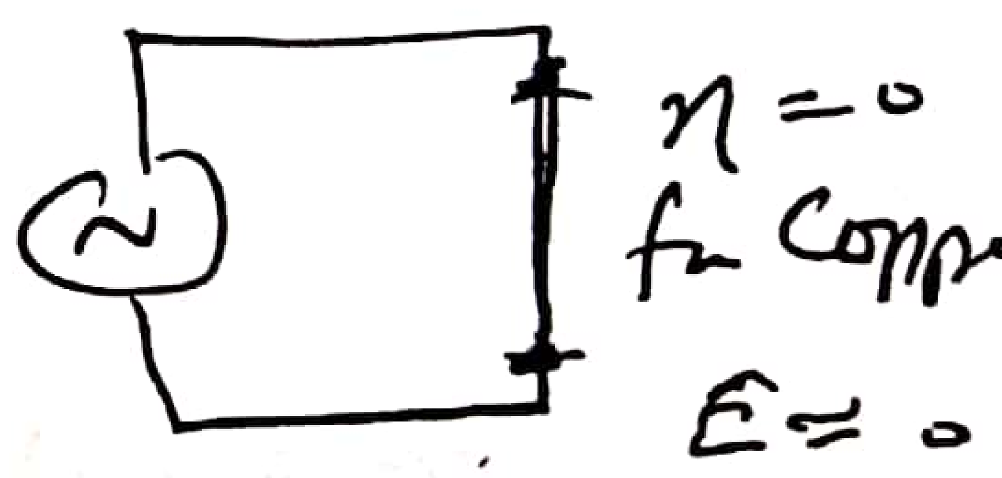


Fig-a Fields propagate in free space

Fig-b Fields vanishes at surface of copper medium

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SNo	Element	Conductivity $\sigma \times 10^{7}$ (S/m)	$\times 10^{-7}$ μ (A/m)	Skin depth δ (m)
1	Silver	6.17	4π	$\frac{0.0642}{\sqrt{f}}$
2)	Copper	5.80	4π	$\frac{0.0660}{\sqrt{f}}$
3	Al	3.72	4π	$\frac{0.0826}{\sqrt{f}}$
4	Brass	1.57	4π	$\frac{0.127}{\sqrt{f}}$

Take an example of Copper 'Cu'

At frequency, $f = 50\,Hz$ for Skin depths,

$$\delta = \frac{0.0642}{\sqrt{50}} = 0.0093\,m = 9\,mm$$

At frequency, $f = 3\,GHz$

$$\delta = \frac{0.0642}{\sqrt{3\times10^{9}}} = \underline{1.2\,\mu m}$$

The impedance at various frequencies

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} = \sqrt{\frac{2\pi f\mu}{\sigma}}$$

For 50 Hz

$$\eta = 2.6\times10^{-6}\,\Omega,$$

For 3 GHz

$$\eta = 0.02\,\Omega$$

Solve δ & η for other elements given in the table