EMWP-S10

Plane Electromagnetic Wave Propagation.

Reflection and Refraction at the boundary of two non-conducting media.

Normal incidence:

Let us a plane EM-Wave travelling along the Z-direction with $\bar{E}_1$ & $\bar{H}_1$ as the incident field vectors travelling in medium-1 with ϵ_1 & μ_1 permittivity & permeability of medium-1 respectively. While $\bar{E}_2$ & $\bar{H}_2$ describe the

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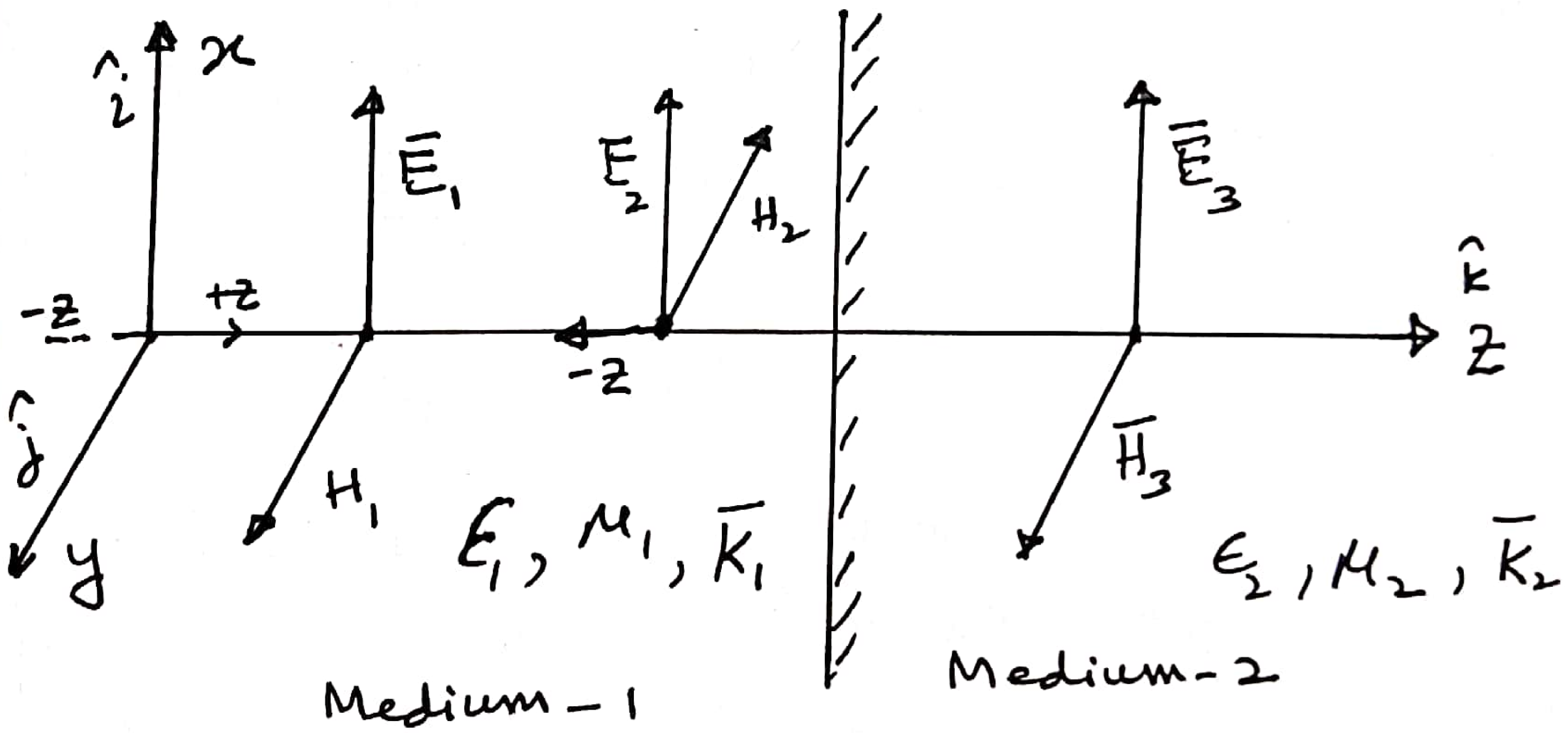

Fig: Reflection and Refraction of plane EM-Waves at the boundary of two non-conducting media

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reflected waves travelling in the -ve of Z-axis. The field vectors $\bar{E}_3$ & $\bar{H}_3$

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represents the transmitted or refracted field vectors in medium-2. Let ϵ_2 & μ_2 be the pemitivity and permeability of the medium-2 respectively.

The interface at $z=0$ is coincident with the xy-plane the electric fields are polarized in the x-direction. $\bar{E}$ is every where parallel to the x-axis and are given as:

$$\left.\begin{aligned} \bar{E}_1 &= \hat{i}\, E_{1,0}\, e^{j(k_1 z-\omega t)} \quad \text{———(a)} \\ \bar{E}_2 &= \hat{i}\, E_{2,0}\, e^{-j(k_1 z+\omega t)} \quad \text{———(b)} \\ \bar{E}_3 &= \hat{i}\, E_{3,0}\, e^{j(k_2 z-\omega t)} \quad \text{———(c)} \end{aligned}\right\} \text{— (1)}$$

In the above equations $\bar{k}_1$ & $\bar{k}_2$ are the wave vectors in the medium-1 & medium-2 respectively. While $E_{1,0}$, $E_{2,0}$ & $E_{3,0}$ are the amplitudes.

The magnetic field associated with the electric field is given as:

$$\bar{B}_{(z,t)} = \sqrt{\epsilon\mu}\ \hat{k} \times \bar{E}_0\, e^{j(kz-\omega t)}$$

or $$\bar{H} = \sqrt{\frac{\epsilon}{\mu}}\ \hat{k} \times \hat{i}\, E_0\, e^{j(kz-\omega t)}$$

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Hence the magnetic field vectors in medium-1 & medium-2 are expressed as;

$$\bar{H}_1 = \hat{j}\sqrt{\frac{\epsilon_1}{\mu_1}}\, E_{1,0}\, e^{j(k_1 z - \omega t)} \quad \text{———— (a)}$$

$$\bar{H}_2 = -\hat{j}\sqrt{\frac{\epsilon_1}{\mu_1}}\, E_{2,0}\, e^{-j(k_1 z - \omega t)} \quad \text{———— (b)} \qquad \text{—(2)}$$

$$\bar{H}_3 = \hat{j}\sqrt{\frac{\epsilon_2}{\mu_2}}\, E_{3,0}\, e^{j(k_2 z - \omega t)} \quad \text{———— (c)}$$

Since the normal components of the field vanishes and only tangential components of the electric and magnetic fields are considered as per boundary conditions for the EM-wave propagation.

Since the medium-1 & medium-2 are non-conducting for which we take conductivities $g_1 = g_2 = 0$ for both media

At the interface $z = 0$ the above equations (1) & (2) are reduced to the following forms:

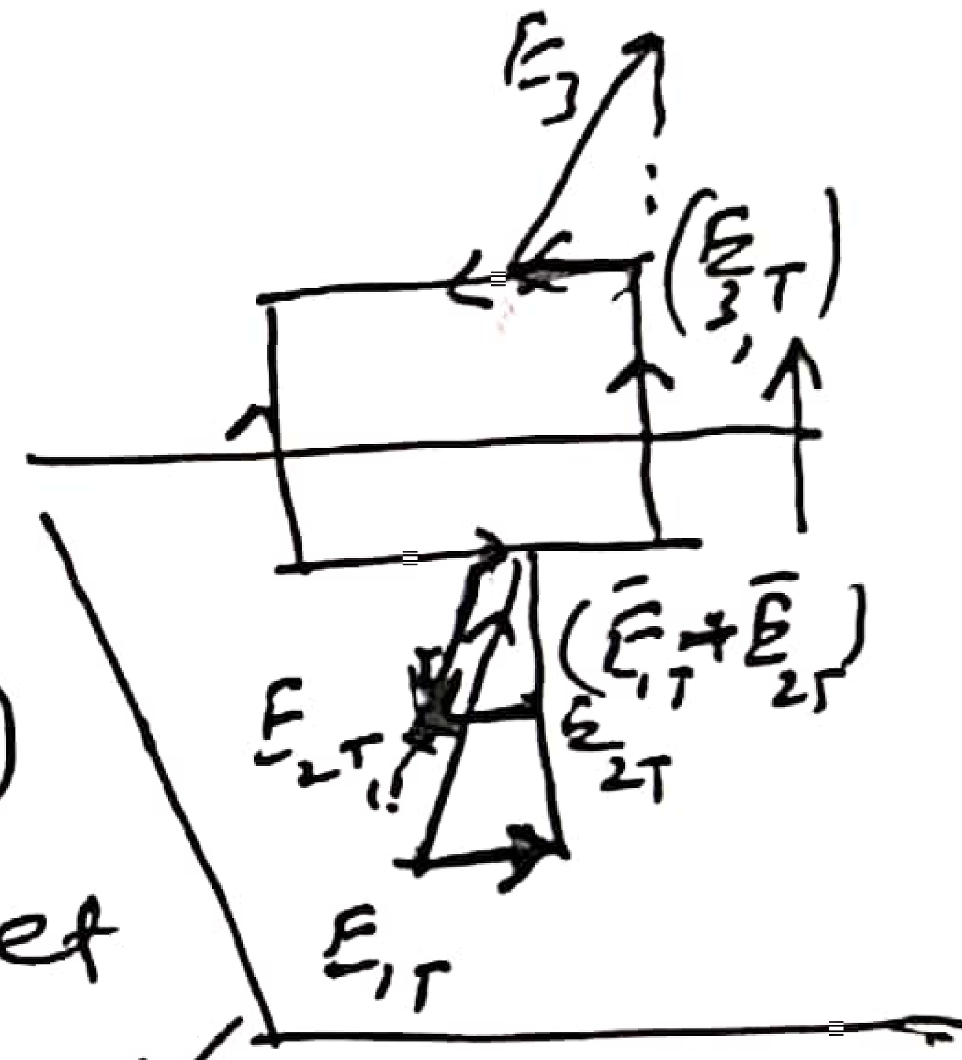
(Continued)

$$\bar{E}_1 = \hat{i}\, E_{1,0}\, e^{-j\omega t} \text{ ---- (a)}$$
$$\bar{E}_2 = \hat{i}\, E_{2,0}\, e^{-j\omega t} \text{ ---- (b)} \quad (1\text{-}a)$$
$$\bar{E}_3 = \hat{i}\, E_{3,0}\, e^{-j\omega t} \text{ ---- c}$$

As boundary conditions $\bar{E}$ field vector, in medium-1 & in medium-2 must be continuous i.e the field vector sum in medium-1 is equal to the field vector in medium-2

Hence

$$\bar{E}_1 + \bar{E}_2 = \bar{E}_3 \text{ ---- } (1\text{-}b)$$



putting the values from eq (1-a) in the above equation we get

$$\hat{i}\, E_{1,0}\, \cancel{e^{-j\omega t}} + \hat{i}\, E_{2,0}\, \cancel{e^{-j\omega t}} = \hat{i}\, E_{3,0}\, \cancel{e^{-j\omega t}}$$

Taking only the magnitude of the amplitudes

$$E_{1,0} + E_{2,0} = E_{3,0} \text{ ---- } 3)$$

Similarly at $z=0$ the magnetic field, wave equations are given as

$$\bar{H}_1 + \bar{H}_2 + \bar{H}_3 = \text{ ---- } (3a)$$

(Continued)

$$\bar{H}_1 = \hat{j}\sqrt{\frac{\epsilon_1}{\mu_1}}\, E_{1,0}\, e^{-j\omega t} \quad \text{—— (a)}$$

$$\bar{H}_2 = -\hat{j}\sqrt{\frac{\epsilon_1}{\mu_1}}\, E_{2,0}\, e^{-j\omega t} \quad \text{—— (b)} \qquad \text{—(2-a)}$$

$$\bar{H}_3 = \hat{j}\sqrt{\frac{\epsilon_2}{\mu_2}}\, E_{3,0}\, e^{j\omega t} \quad \text{—— c}$$

In the above equations $\hat{j}$ is the unit vector for y-axis while "j in the exponent term "$e^{-j\omega t}$" is the complex "$j=\sqrt{-1}$" also called j-operator in the complex vector algebra.

putting the above terms of equation (2a) in the equation (3a) we get

$$\hat{j}\sqrt{\frac{\epsilon_1}{\mu_1}}\,(E_{1,0} - E_{2,0})\, \cancel{e^{-j\omega t}} = \hat{j}\sqrt{\frac{\epsilon_2}{\mu_2}}\, E_{3,0}\, \cancel{e^{-j\omega t}}$$

Considering the magnitude terms only ;

$$\sqrt{\frac{\epsilon_1}{\mu_1}}\,(E_{1,0} - E_{2,0}) = \sqrt{\frac{\epsilon_2}{\mu_2}}\, E_{3,0} \quad \text{—(4)}$$

Let us substitute the $E_{3,0}$ from equation (3) as $E_{3,0} = E_{1,0} + E_{2,0}$ in eq (4) & re-arranging

$$\sqrt{\frac{\epsilon_1}{\mu_1}}\, E_{1,0} + \sqrt{\frac{\epsilon_1}{\mu_1}}\, E_{2,0} = \sqrt{\frac{\epsilon_2}{\mu_2}}\, E_{1,0} + \sqrt{\frac{\epsilon_2}{\mu_2}}\, E_{2,0}$$

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$$E_{1,0}\left[\sqrt{\frac{\epsilon_1}{\mu_1}}-\sqrt{\frac{\epsilon_2}{\mu_2}}\right]=E_{2,0}\left(\sqrt{\frac{\epsilon_1}{\mu_1}}+\sqrt{\frac{\epsilon_2}{\mu_2}}\right)$$

$$E_{2,0}=E_{1,0}\,\frac{\sqrt{\frac{\epsilon_1}{\mu_1}}-\sqrt{\frac{\epsilon_2}{\mu_2}}}{\sqrt{\frac{\epsilon_1}{\mu_1}}+\sqrt{\frac{\epsilon_2}{\mu_2}}} \quad \text{———} (5)$$

Now putting $E_{2,0} = E_{3,0} - E_{1,0}$ from equation (3) in equation (4) we have

$$\sqrt{\frac{\epsilon_1}{\mu_1}}E_{1,0}-\sqrt{\frac{\epsilon_1}{\mu_1}}E_{3,0}+\sqrt{\frac{\epsilon_1}{\mu_1}}E_{1,0}=\sqrt{\frac{\epsilon_2}{\mu_2}}E_{3,0}$$

Re-arranging the above equations

$$2\sqrt{\frac{\epsilon_1}{\mu_1}}E_{1,0}=E_{3,0}\left(\sqrt{\frac{\epsilon_1}{\mu_1}}+\sqrt{\frac{\epsilon_2}{\mu_2}}\right)$$

or

$$E_{3,0}=E_{1,0}\left[\frac{2\sqrt{\frac{\epsilon_1}{\mu_1}}}{\sqrt{\frac{\epsilon_1}{\mu_1}}+\sqrt{\frac{\epsilon_2}{\mu_2}}}\right] \quad \text{———} (6)$$

(Contd.)

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The equations (5) & (6) determine the electric field of the reflected and transmitted/refracted in terms of the incident field waves.

If the values of these equations are substituted in equations ① & ② we have vector field terms

$$\bar{E}_2 = \hat{i}\, E_{1,0} \left[\frac{\sqrt{\frac{\epsilon_1}{\mu_1}} - \sqrt{\frac{\epsilon_2}{\mu_2}}}{\sqrt{\frac{\epsilon_1}{\mu_1}} + \sqrt{\frac{\epsilon_2}{\mu_2}}} \right] e^{-j(k_1 z + \omega t)} \qquad \text{—(a)}$$

$$\bar{E}_3 = \hat{i}\, E_{1,0} \left[\frac{2\sqrt{\frac{\epsilon_1}{\mu_1}}}{\sqrt{\frac{\epsilon_1}{\mu_1}} + \sqrt{\frac{\epsilon_2}{\mu_2}}} \right] e^{j(k_2 z - \omega t)} \qquad \text{—(b)} \qquad \text{—(7)}$$

$$\bar{E}_1 = \hat{i}\, E_{1,0}\, e^{i(k_1 z - \omega t)} \qquad \text{—c}$$

Similarly for Magnetic field vectors

$$\bar{H}_1 = \hat{j} \sqrt{\frac{\epsilon_1}{\mu_1}}\, E_{1,0}\, e^{j(k_1 z - \omega t} \qquad \text{—(a)}$$

$$\bar{H}_2 = -\hat{j}\, E_{1,0} \sqrt{\frac{\epsilon_1}{\mu_1}} \left[\frac{\sqrt{\frac{\epsilon_1}{\mu_1}} - \sqrt{\frac{\epsilon_2}{\mu_2}}}{\sqrt{\frac{\epsilon_1}{\mu_1}} + \sqrt{\frac{\epsilon_2}{\mu_2}}} \right] e^{-j(k_1 z + \omega t)} \qquad \text{—(b)} \qquad (8)$$

(Contd.)

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$$\bar{H}_3 = 2\hat{j}\, E_{1,0}\left[\frac{\left(\sqrt{\frac{\epsilon_1}{\mu_1}}\right)\left(\sqrt{\frac{\epsilon_2}{\mu_2}}\right)}{\sqrt{\frac{\epsilon_1}{\mu_1}}+\sqrt{\frac{\epsilon_2}{\mu_2}}}\right] e^{j(k_2 z-\omega t)} \quad (c) \text{---} (8)$$

Refractive Index: "n"

The refractive index of a medium is defined is, it is the ratio between the velocity / speed of light in vacuum to the speed of the wave in the medium through which it travells. It is denoted by "n". So $n = \frac{c}{u}$ (in vacuum) where "c" is the speed of light & "u" is the phase velocity in the medium.

$$n = \frac{\text{Speed of light in vacuum}}{\text{Speed of wave in the medium}}$$

for optically transparent medium

$$\mu_1 = \mu_2 = \mu_0$$

For free space or vacuum $\epsilon_1 = \epsilon_0 \epsilon_{r_1}$

$\epsilon_2 = \epsilon_0 \epsilon_{r_2}$ & $\epsilon_3 = \epsilon_0 \epsilon_{r_3}$

The phase velocity $n = \frac{1}{\sqrt{\epsilon\mu}} = \frac{1}{\sqrt{\epsilon\mu_0}}$ As $\mu = \mu_0$

While $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$