EMWP-S11

Plane Electromagnetic Wave Propagation

Reflection from a Conducting plane:

Normal incidence:

Let us consider that a plane wave is propagating along the +Z-direction. This plane EM-Wave is travelling in a non-conducting medium and incident on a conducting plane normal to the surface of the conducting plane.

let $\bar{E}_1 = \hat{i}\, E_{1,0}\, e^{j(k_1 z - \omega t)}$ be the incident $\bar{E}$ field function and

$\bar{H}_1 = j\sqrt{\frac{\epsilon_1}{\mu_1}}\, E_{1,0}\, e^{j(k_1 z - \omega t)}$ be the incident Magnetic field intensity.

let $\bar{E}_2 = \hat{i}\, E_{2,0}\, e^{-j(k_1 z - \omega t)}$

& $\bar{H}_2 = -j\sqrt{\frac{\epsilon_1}{\mu_1}}\, E_{2,0}\, e^{-j(k_1 z + \omega t)}$

be the reflected Electric and Magnetic field vectors. When $\bar{K}$ is a vector & given as $K = \omega\sqrt{\epsilon\mu}$

The wave in the conducting medium has the following refracted $\bar{E}_3$ & $\bar{H}_3$ field vectors

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$$\bar{E}_3 = \hat{i}\, E_{3,0}\, e^{j(\gamma_2 z - \omega t)}$$

$$\bar{H}_3 = \hat{j}\, \frac{\gamma_2}{\omega \mu_2}\, E_{3,0}\, e^{j(\gamma_2 z - \omega t}$$

Where $\gamma_2 = \alpha_2 + j\beta_2$

Where g_2 is the conductivity of the conducting medium

& α_2 & β_2 are given as

$$\alpha_2 = \pm \omega \sqrt{\epsilon_2 \mu_2} \left[\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{g_2^2}{\omega^2 \epsilon_2^2}}\right]^{1/2}$$

& $\beta_2 = \dfrac{\omega g_2 \mu_2}{2\alpha_2}$

The continuity of the tangential components $\bar{E}$ & $\bar{H}$ gives the following equations

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$$E_{1,0} + E_{2,0} = E_{3,0} \quad \text{---} (1)$$

& $$H_{1,0} + H_{2,0} = H_{3,0} \quad \text{---} (2)$$

putting $H = \sqrt{\frac{\epsilon}{\mu}} E$ in equation (2)
with their respective ϵ & μ values
i.e $H_{1,0} = \sqrt{\frac{\epsilon_1}{\mu_1}} E_{1,0}$, $H_{2,0} = -\sqrt{\frac{\epsilon_1}{\mu_1}} E_{2,0}$

& $H_{3,0} = \frac{\gamma_2}{\omega\mu_2} E_{3,0}$ putting in equ:(2)

$$\sqrt{\frac{\epsilon_1}{\mu_1}} E_{1,0} - \sqrt{\frac{\epsilon_1}{\mu_1}} E_{2,0} = \frac{\gamma_2}{\omega\mu_2} E_{3,0} \quad \text{---} (3)$$

Solving equations (1) & (3),
putting the values of $E_{3,0}$ from equ: (1)
in equation (3).
After rearranging and simplifying,
we get

$$E_{2,0} = E_{1,0} \left[\frac{1 - \frac{\gamma_2}{\omega\mu_2}\sqrt{\frac{\mu_1}{\epsilon_1}}}{1 + \frac{\gamma_2}{\omega\mu_2}\sqrt{\frac{\mu_1}{\epsilon_1}}} \right] \quad \text{---} (4)$$

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& Similarly now putting the value of $E_{2,0}$ from equation (1) in eqn (3)

i.e $E_{2,0} = (E_{3,0} - E_{1,0})$

∴ After putting the values, rearranging and simplifying we get

$$E_{3,0} = \frac{2E_{1,0}}{1 + \left(\frac{\gamma_2}{\omega\mu_2}\right)\left(\sqrt{\frac{\mu_1}{\epsilon_1}}\right)} \quad \text{—— (5)}$$

For an infinite conducting plane the conductivity $g = \infty$ & the term in the denominator contains "γ_2" where

$$\gamma_2 = \alpha_2 + j\beta_2$$

$$= \pm\omega\sqrt{\epsilon_2\mu_2}\left[\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{g_2^2}{\omega^2\epsilon_2^2}}\right]^{1/2} + j\left(\frac{\omega g_2 \mu_2}{2\alpha_2}\right)$$

Since $g_2 \to \infty$ so as $\gamma_2 \to \infty$

& $\frac{1}{\infty} \to 0$ so we express the equation (5) as

$E_{3,0} = 0$ means transmitted wave amplitude is zero

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By putting $E_{3,0} = 0$
in equation(1) we get

$$E_{2,0} = -E_{1,0} \quad \text{—— (6)}$$

This shows that all of the incident energy is reflected and no energy is penetrated.

Now let us take good conductor for which g is not infinity but still large enough that

$\frac{g_2}{\omega \epsilon_2} >> 1$ putting in equation of α_2 we get

$$\alpha_2 = \pm \omega \sqrt{\epsilon_2 \mu_2} \left[\cancel{\tfrac{1}{2}} - \frac{1}{2}\sqrt{\cancel{1} + \frac{g_2^2}{\omega^2 \epsilon_2^2}} \right]^{1/2}$$

$$\alpha_2 = \pm \omega \sqrt{\epsilon_2 \mu_2} \left[\pm \frac{g_2}{2\omega\epsilon_2} \right]^{1/2}$$ simplify it

$$\alpha_2 = \sqrt{\frac{\omega^{\cancel{2}} \cancel{\epsilon_2} \mu_2 \times g_2}{2 \cancel{\omega} \cancel{\epsilon_2}}} = \sqrt{\frac{\omega g_2 \mu_2}{2}}$$

$$\beta_2 = \frac{\omega g_2 \mu_2}{2\alpha_2} = \frac{\omega g_2 \mu_2}{2\sqrt{\frac{\omega g_2 \mu_2}{2}}}$$

(contd:)

$$\beta_2 = \sqrt{\frac{\omega g_2 \mu_2}{2}}$$

Now putting α_2 & β_2 values in $\gamma_2 = \alpha_2 + j\beta_2$

$$\gamma_2 = \sqrt{\frac{\omega \mu_2 g_2}{2}} + j\sqrt{\frac{\omega \mu_2 g_2}{2}}$$

$$\gamma_2 = (1+j)\sqrt{\frac{\omega \mu_2 g_2}{2}}$$

Now puttn "γ_2" in eqn: (4) we have

$$E_{2,0}' = \frac{1-(1+j)\sqrt{\frac{\omega\mu_2 g_2}{2}}\left(\frac{1}{\omega\mu_2}\right)\sqrt{\frac{\mu_1}{\epsilon_1}}}{1+(1+j)\sqrt{\frac{\omega\mu_2 g_2}{2}}\left(\frac{1}{\omega\mu_2}\right)\sqrt{\frac{\mu_1}{\epsilon_1}}}(E_{1,0})$$

$$E_{2,0} = \frac{1-(1+j)\sqrt{\frac{g_2\mu_1}{2\omega\mu_2\epsilon_1}}}{1+(1+j)\sqrt{\frac{g_2\mu_1}{2\omega\mu_2\epsilon_1}}}(E_{1,0}) \quad -(7)$$

Since $E_{2,0}$ is a complex equation This shows that a phase difference from $(0-\pi)$ is possible for the reflected wave.