EMWP-S12 Electromagnetic Waves

Reflection & Transmission Coefficients R_n & T_n & To Show that $R_n + T_n = 1$, $(\mu_1 = \mu_2 = \mu_0)$

(For optically transparent media), (Non-conducting media)

The reflection coefficient R_n is defined as the ratio of average reflected poynting vector to average incident poynting vector absolute values. $R_n = \frac{S_{2av}}{S_{1av}}$ —— (1)

Where $S_{2(av)} = |\frac{1}{2} \bar{E}_2 \times \bar{H}_2|$ is the av: reflected poynting vector

& $S_{1(av)} = |\frac{1}{2} \bar{E}_1 \times \bar{H}_1|$, is the av: incident poynting vector

The transmission coefficient "T_n" is defined as the ratio of the average transmitted poynting vector to the average incident poynting vector with absolute values.

Let $S_{3(av)} = |\frac{1}{2} \bar{E}_3 \times \bar{H}_3|$ be the average reflected poynting vector

(contd:)

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and $S_{1(av)} = \left|\frac{1}{2}\bar{E}_1 \times \bar{H}_1\right|$ as already mentioned be the average incident Poynting vector.

$$T_n = \frac{S_{3av}}{S_{1av}} \quad \text{—— (2)}$$

Let us solve the relations for R_n & T_n one by one.

Reflection coefficient (R_n):

$$R_n = \left(\frac{S_2}{S_1}\right)_{av} = \left|\frac{\frac{1}{2}\bar{E}_2 \times \bar{H}_2}{\frac{1}{2}\bar{E}_1 \times \bar{H}_1}\right|$$

$$R_n = \left|\frac{\bar{E}_2 \times \bar{H}_2}{E_1 \times \bar{H}_1}\right| \quad \text{—— (3)}$$

$$|\bar{E}_1 \times \bar{H}_1| = ? \qquad \text{Since } \bar{E}_1 = \hat{i}\, E_{1,0}\, e^{-j\omega t}$$

$$\bar{H}_1 = \hat{j}\, E_{1,0}\, e^{-j\omega t}\left(\sqrt{\frac{\epsilon_1}{\mu_1}}\right)$$

$$|\bar{E}_1 \times \bar{H}_1| = \left|\hat{i}\times\hat{j}\; E_{1,0}^2 \left(\sqrt{\frac{\epsilon_1}{\mu_1}}\right)(e^{-j\omega t})\cdot(e^{-j\omega t})\right|$$

$$\hat{i}\times\hat{j} = \hat{k} \quad \& \quad |\hat{k}| = 1$$

Also $$|e^{-j\omega t}| = |\cos(\omega t) - j\sin(\omega t)|$$

$$= \sqrt{\cos^2\omega t + \sin^2\omega t}$$

$$|e^{j\omega t}| = \sqrt{1} = 1$$

So we have $$|E_1 \times H_1| = \sqrt{\frac{\epsilon_1}{\mu_1}}\, E_{1,0}^2 \quad \text{—— (4)}$$

(Contd.)

Now for $|\bar{E}_2 \times \bar{H}_2| = ?$

$$\bar{E}_2 = \hat{i}\, E_{2,0}\, e^{-j\omega t}$$
$$\bar{H}_2 = -\hat{j}\sqrt{\frac{\epsilon_1}{\mu_1}}\, E_{2,0}\, e^{-j\omega t}$$

$$|\bar{E}_2 \times \bar{H}_2| = \left| -(\hat{i}\times\hat{j})\left(\sqrt{\frac{\epsilon_1}{\mu_1}}\right) E_{2,0}^2\, e^{-j\omega t} \cdot e^{-j\omega t} \right|$$

again $\hat{i}\times\hat{j} = \hat{k}$ & $|\hat{k}| = 1$

also $|e^{-j\omega t}| = 1$, $|-1| = +1$

So we have

$$|\bar{E}_2 \times \bar{H}_2| = \sqrt{\frac{\epsilon_1}{\mu_1}}\, E_{2,0}^2 \quad \text{--- (5)}$$

Now putting (4) & (5) in equation (3). we get

$$R_n = \frac{\cancel{\sqrt{\frac{\epsilon_1}{\mu_1}}}\, E_{2,0}^2}{\cancel{\sqrt{\frac{\epsilon_1}{\mu_1}}}\, E_{1,0}^2} = \left(\frac{E_{2,0}}{E_{1,0}}\right)^2$$

Hence the reflection coefficient can also be defined as the ratio of the squared reflected to incident amplitudes of the EM-Waves.

We can express R_n in terms of Refractive indices as we know $n_1 = \sqrt{\frac{\epsilon_1}{\epsilon_0}}$ & $n_2 = \sqrt{\frac{\epsilon_2}{\epsilon_0}}$

$$\left(\frac{E_{2,0}}{E_{1,0}}\right)^2 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

(Contd.)

Transmission Coefficient "T_n":

Since $T_n = \left(\frac{S_3}{S_1}\right)_{av} = \left|\frac{\frac{1}{2}\bar{E}_3 \times \bar{H}_3}{\frac{1}{2}\bar{E}_1 \times \bar{H}_1}\right|$ — (6)

Lets solve $|\bar{E}_3 \times \bar{H}_3| = ?$

Since $\bar{E}_3 = \hat{i}\, E_{3,0}\, e^{-j\omega t}$

& $\bar{H}_3 = \hat{j}\sqrt{\frac{\epsilon_2}{\mu_2}}\, E_{3,0}\, e^{-j\omega t}$

So we have

$$|E_3 \times H_3| = \left|(\hat{i}\times\hat{j})\sqrt{\frac{\epsilon_2}{\mu_2}}\, E_{3,0}^2 \cdot e^{-j\omega t} \cdot e^{-j\omega t}\right|$$

since $\hat{i}\times\hat{j} = \hat{k}$ & $|\hat{k}| = 1$

Also $|e^{-j\omega t}| = 1$

So $|\bar{E}_3 \times \bar{H}_3| = \sqrt{\frac{\epsilon_2}{\mu_2}}\left(E_{3,0}^2\right)$ — (7)

Now putting the values of equation (7) & (4)

we get
$$T_n = \frac{\sqrt{\frac{\epsilon_2}{\mu_2}}}{\sqrt{\frac{\epsilon_1}{\mu_2}}}\left(\frac{E_{3,0}}{E_{1,0}}\right)^2$$

$\mu_1 = \mu_2 = \mu_0$

$$T_n = \frac{\sqrt{\frac{\epsilon_2}{\mu_0}}}{\sqrt{\frac{\epsilon_1}{\mu_0}}}\left(\frac{E_{3,0}}{E_{1,0}}\right)^2 = \sqrt{\frac{\epsilon_2}{\epsilon_1}}\left(\frac{E_{3,0}}{E_{1,0}}\right)^2$$

$n_1 = \sqrt{\frac{\epsilon_1}{\epsilon_0}}$

$n_2 = \frac{\epsilon_2}{\epsilon_0}$

$\frac{n_2}{n_1} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$

$$T_n = \left(\frac{n_2}{n_1}\right)\left(\frac{E_{3,0}}{E_{1,0}}\right)^2$$

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As $\left(\frac{E_{3,0}}{E_{1,0}}\right) = \left(\frac{2n_1}{n_1+n_2}\right)$

$$T_n = \frac{n_2}{n_1}\left(\frac{2n_1}{n_1+n_2}\right)^2 \quad \text{—— (8)}$$

Hence the Transmission Coefficient is expressed in terms of the ratio of the Squared transmitted to reflected amplitudes of the EM-waves.

Now to show that $R_n + T_n = 1$

As we $R_n = \left(\frac{n_1-n_2}{n_1+n_2}\right)^2$ —— (1)

& $T_n = \frac{n_2}{n_1}\left(\frac{n_1-n_2}{n_1+n_2}\right)^2$ —— (2)

$$R_n + T_n = \left(\frac{n_1-n_2}{n_1+n_2}\right)^2 + \frac{n_2}{n_1}\left(\frac{2n_1}{n_1+n_2}\right)^2$$

$$= \frac{(n_1-n_2)^2}{(n_1+n_2)^2} + \frac{n_2}{\cancel{n_1}}\frac{(4n_1^{\cancel{2}})}{(n_1+n_2)^2}$$

$$= \frac{n_1^2+n_2^2-2n_1n_2}{(n_1+n_2)^2} + \frac{4n_1n_2}{(n_1+n_2)^2}$$

$$= \frac{1}{(n_1+n_2)^2}\left(n_1^2+n_2^2-2n_1n_2+4n_1n_2\right)$$

$$= \frac{1}{(n_1+n_2)^2}\left(n_1^2+n_2^2+2n_1n_2\right)$$

$$R_n + T_n = \frac{1}{\cancel{(n_1+n_2)^2}} \times \cancel{(n_1+n_2)^2} = 1$$