EMWP 13

Electromagnetic Wave Propagation
Boundary Conditions for $\bar{B}, \bar{H}, \bar{D}$ & $\bar{E}$ fields across The interface of two non-conducting media.

(i) Boundary Conditions for $\bar{B}$:

Normal Component of $\bar{B}$:-

Let us consider That magnetic field $\bar{B}_1$ vector incident on the interface of two media ① & ② with μ_1 & μ_2 respective permeabilities.

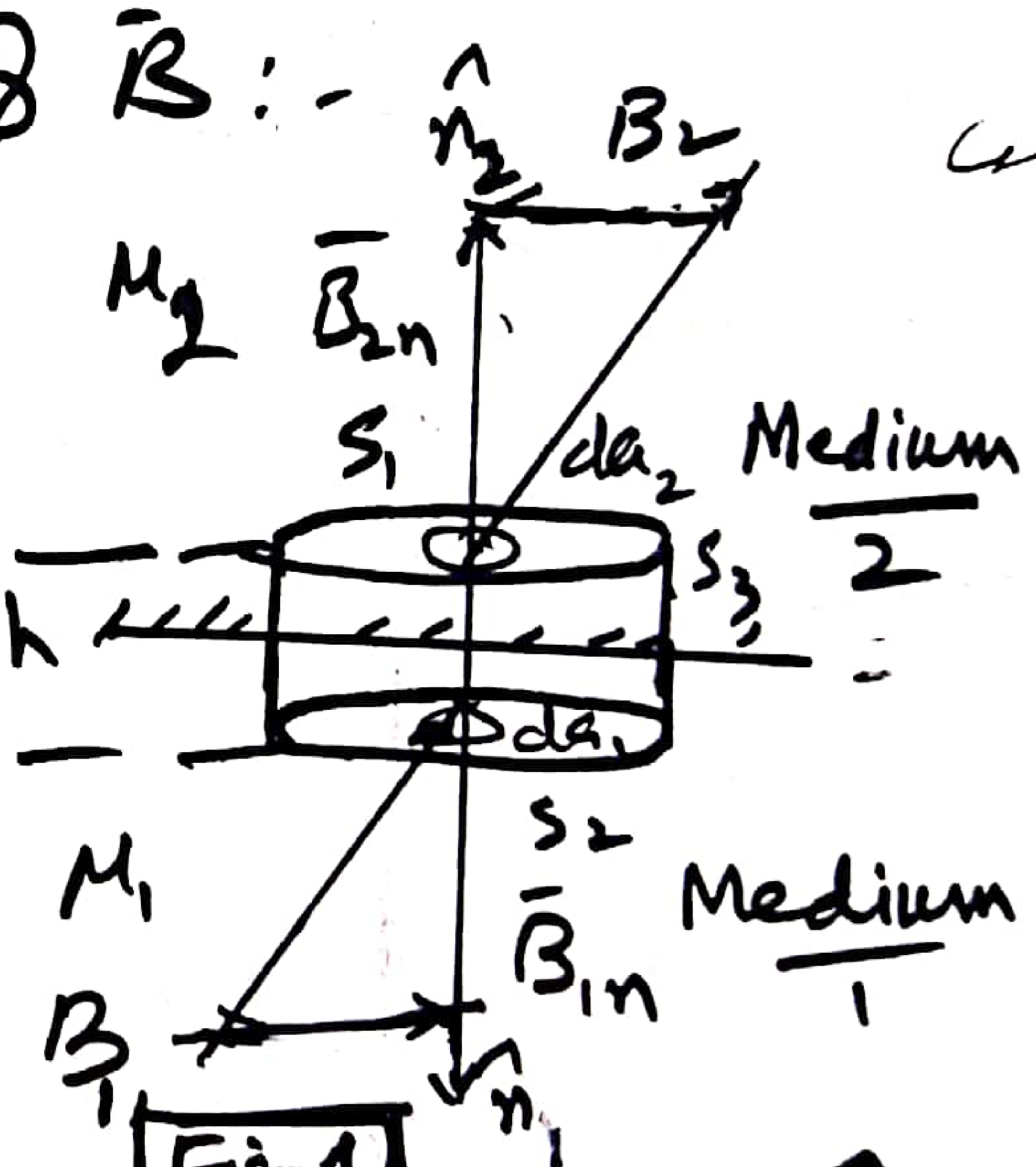

Fig-1

let $\bar{B}_2$ be The Magnetic field vector in medium ②, while $\hat{n}_1$ & $\hat{n}_2$ are the normal unit vectors of medium-1 and medium-2 respectively. as Shown in Fig-1

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Let us consider a small pill box on the interface with negligible height "h" and surface S_1 & S_2 of the pill box cylinder. 2/10

The surface "S_3" of the pill box is also considered negligible due to the very small height.

We picked the pill box cylinderical surface to apply the Gauss's Law of flux

$$\phi_m = \int_S \bar{B} \cdot \hat{n}\, da = 0 \quad \text{——(1)}$$

Since the Gaussian surface consists of two main surfaces S_1 & S_2. Hence the magnetic flux through surfaces across the interface is expressed as

$$\phi_m = \int_{S_1} \bar{B}_1 \cdot \hat{n}_1\, da_1 + \int_{S_2} \bar{B}_2 \cdot \hat{n}_2\, da_2 = 0$$

$$= \bar{B}_1 \cdot \hat{n}_1 \int_{S_1} da_1 + \bar{B}_2 \cdot \hat{n}_2 \int_{S_2} da_2 = 0$$

$$\int_{S_1} da_1 = S_1 \;,\; \& \int_{S_2} da_2 = S_2 \;,\; S_1 = S_2 = S$$

$$= \bar{B}_{1n} S + \bar{B}_{2n} S = 0 \quad \text{——(2)}$$

$$\bar{B}_{1n} = \bar{B}_{2n} \quad \text{——(3)}$$

(Contd.)

Hence the normal component of $\bar{B}$ is continuous across the interface of media.

(ii) Tangential Component of $\vec{H}$:

The relationship for the magnetic field intensity $\vec{H}$ may be expressed in Ampere's Law

$$\oint_l \bar{H} \cdot d\bar{l} = I \quad \text{—— (1)}$$

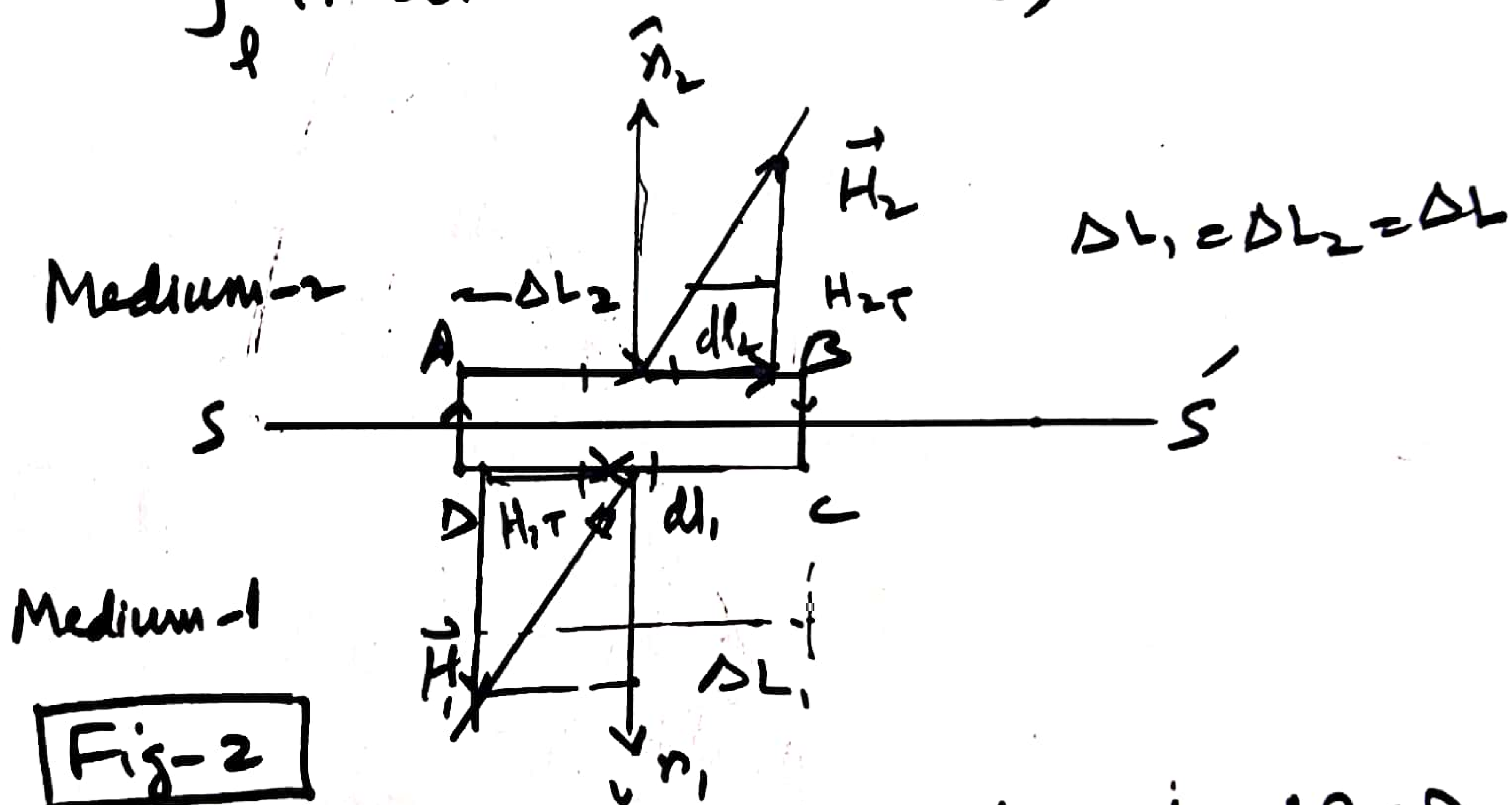

Fig-2

Let us consider a closed path ABCD across the interface. The segment $\bar{AB}$ & $\bar{CD}$ are large as compared to the $\bar{AD}$ & $\bar{BC}$.

$\bar{AB} = \Delta L_1$ & $\bar{CD} = \Delta L_2$

The segments $\bar{AB}$ = ΔL_1 & ... small length while dl_1 & dl_1 are the small length elements of segments ΔL_1 & ΔL_2 respectively, as shown in fig-2.

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Appling Ampere's Law to the loop

$$\oint_l \bar{H}\cdot d\bar{l} = \int_l \bar{H}_1 \cdot d\bar{l}_1 + \int_l \bar{H}_2 \cdot d\bar{l}_2$$

Since the tangential Component of $\bar{H}_{1T} \uparrow\uparrow dl_1$ & $\bar{H}_{2T} \uparrow\downarrow d\bar{l}_2$ Hence we have

$$= H_{1T}\int_l dl_1 - H_2\int_l dl_2$$

$$= H_{1T}\,\Delta L_1 - H_2\,\Delta L_2$$

Since $\Delta L_1 = \Delta L_2 = \Delta L$

$$\oint_R \bar{H}\cdot d\bar{l} = H_{1T}\,\Delta L - H_{2T}\,\Delta L = I$$

If the interface is having zero free charges Then $I = 0$

So $H_{1T}\,\cancel{\Delta L} - H_{2T}\,\cancel{\Delta L} = 0$

or $H_{1T} = H_{2T}$ ——— (2)

The tangential Component of $\bar{H}$ will be Continuous. But if interface contains free charges then

$$H_{1T} - H_{2T} = \frac{I}{\Delta L} \text{ ——— (3)}$$

Note:

or $H_{1t} - H_{2t} = \frac{I}{\Delta L}$ $\quad \hat{n}\times(\bar{H}_{1t} - \bar{H}_{2t})$

Normal Component of $\bar{D}$:-

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Let us Consider $\bar{D}_1$ field vector incident at the interface of two media Medium-1 & medium-2 $\bar{D}_1$ be in the medium while $\bar{D}_2$ field vector

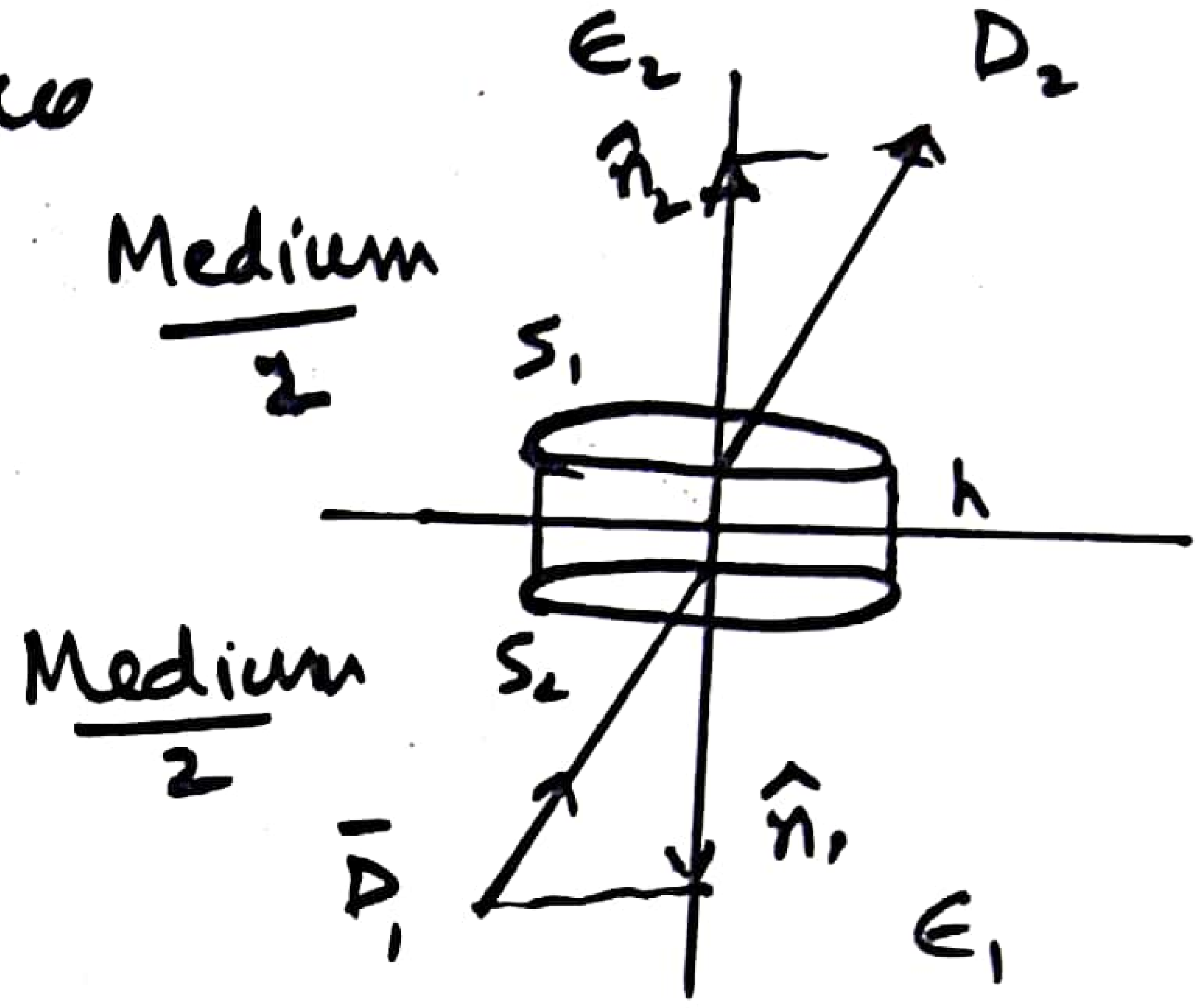

Fig-3

propagates in medium-2. Let ϵ_1 & ϵ_2 be the permitivities of medium-1 & medium-2 respectively, as shown in (Fig-3).

Let S_1 & S_2 be the surfaces of a pill box on the interface with negligible height "h". Let $\hat{n}_1$ & $\hat{n}_2$ be the two normal unit vectors in medium-1 and medium-2 respectively.

From Maxwell's first equation in integral form we have

$$\int_V \bar{\nabla}\cdot\bar{D}\,dv = \int_V \rho\,dv \quad \text{—— (1)}$$

Applying Gauss's divergence Theorem to equation (1) where ρ is the volume charge-density.

Contd.

$$\int_s \bar{D}\cdot\hat{n}\, da = \int_s \sigma\, da \quad -(2)$$

where σ is the surface charge density.

$$\int_{s_1} \bar{D}_1\cdot\bar{n}_1\, da - \int_{s_2} \bar{D}_2\cdot\bar{n}_2\, da = \sigma\int_s da$$

where. $|\hat{n}|_1 = n_1 \qquad |\hat{n}|_2 = -\hat{n}_2$

Medium-1 Medium-2

$$\bar{D}_1\cdot\hat{n}_1 \int_{s_1} da - \bar{D}_2\cdot\bar{n}_2\int_{s_2} da = \sigma\int_s da$$

$$\int_{s_1} da = S_1 \quad \& \int_{s_2} da = S_2, \quad \int_s da = S$$

also $S_1 = S_2 = S$

$$(\bar{D}_1\cdot\hat{n}_1)\cancel{S} - (\bar{D}_2\cdot\hat{n}_2)\cancel{S} = \sigma\cancel{S}$$

$$\overline{D_{1n}} - \overline{D_{2n}} = \sigma \quad -(3)$$

If $\sigma = 0$ at the interface

then $\bar{D}_{1n} - \bar{D}_{2n} = 0$

$$\overline{D_{1n}} = \overline{D_{2n}} \quad -(4)$$

Equation (4) shows that the normal component of $\bar{D}$ is continuous across the boundary

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If $\sigma \neq 0$ then the normal component of $\bar{D}$ is not continuous across the interface.

We have the equation of continuity given as

$$\nabla \cdot \bar{J} = -\frac{\partial \rho}{\partial t} \quad \text{—— (5)}$$

integrating both sides of eq (5) over the volume 'v'

$$\int_v \bar{\nabla} \cdot \bar{J} = -\frac{\partial}{\partial t} \int_v \rho \, dv \quad \text{—— (6)}$$

Applying divergence theorem to the L.H.S we have

$$\int_s \bar{J} \cdot \hat{n} da = -\frac{\partial}{\partial t} \int_s \sigma \, da \quad \text{—— (7)}$$

$$\int_s \bar{J} \cdot n da = \int_{s_1} \bar{J}_1 \cdot \hat{n}_1 da - \int_{s_2} \bar{J}_2 \cdot \hat{n}_2 da$$

$$\int_{s_1} da = S_1 \, , \int_{s_2} da_2 = S_2 \, , \int_s da = S$$

$$\& \; S_1 = S_2 = S$$

$$= (\bar{J}_1 \cdot \bar{n}_1) S - (\bar{J}_2 \cdot \hat{n}_2) S \quad \text{putting in (7)}$$

$$J_{1n} S - J_{2n} S = -\frac{\partial \sigma S}{\partial t}$$

$$J_{1n} - J_{2n} = -\frac{\partial \sigma}{\partial t} \quad \text{—— (8)}$$

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If monochromatic radiation is considered

Then surface charge density varies a $e^{-j\omega t}$

$$\sigma = \sigma_0 e^{-j\omega t}$$

$$\frac{\partial}{\partial t}(\sigma) = \frac{\partial}{\partial t}(\sigma_0 e^{-j\omega t}) = -j\omega \sigma_0 e^{-j\omega t}$$

$$\frac{\partial}{\partial t}(\sigma) = -j\omega\sigma \text{ —— } (9)$$

Also $J = g E$ putting in eq (8)

$$g_1 E_{1n} - g_2 E_{2n} = \cancel{-}(j\omega\sigma) \text{ —— } (10)$$

From eq (3) $\bar{D}_{1n} - \bar{D}_{2n} = \sigma$ putting in equation (10)

$$g_1 E_{1n} - g_2 E_{2n} = j\omega(D_{1n} - D_{2n})$$

$$g_1 E_{1n} - g_2 E_{2n} = j\omega \epsilon_1 E_{1n} - j\omega E_{2n}\epsilon_2$$

Rearranging the above equation

$$g_1 E_{1n} - g_2 E_{2n} = j\omega(\epsilon_1 E_{1n} - \epsilon_2 E_{2n})$$

or $E_{1n}(g_1 - j\omega\epsilon_1) = E_{2n}(g - j\omega\epsilon_2)$ — (11)

Equation is the shows the discontinuity of the Normal component of E