Electrostatics: Electric Charge; Conductors and Insulators; Coulomb's Law; Electric Fields due to a Point Charge and an Electric Dipole; Electric Field due to a Charge Distribution; Electric Dipole in an Electric Field; Electric Flux; Gauss' Law and its Applications in Planar; Spherical and Cylindrical Symmetry

Electric Field due to Many (*n*) Point Charges

We can quickly find the net, or resultant, electric field due to more than one point charge. If we place a positive test charge q_0 near n point charges $q_1, q_2, \ldots, q_n$, then the total net force from the n point charges acting on the test charge is

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \cdots + \vec{F}_{0n}.$$

Let are the „n" point charges which is at distances respectively

The net electric field at the position of the test charge is

$$\vec{E} = \frac{\vec{F}_0}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \cdots + \frac{\vec{F}_{0n}}{q_0} = \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n.$$

Let q1,qq2,q3,…………,qn are the n point charges which is at distance r1,r,.r3,……………, rn.

We want to find out the expression of electric field intensity due to assembly of n point charges at field point P.

Then, the total electric field intensity due to assembly of „n" point charges will be

E = E1 + E2 +……….+ En

E1 = Electric Field Intensity at a Field Point due to Point Charge q1 = $\frac{Kq_1}{r_2}\hat{r}_1$

E2 = Electric Field Intensity at a Field Point due to Point Charge q2 = $\frac{Kq_2}{r_2}\hat{r}_2$

E3 = Electric Field Intensity at a Field Point due to Point Charge q3 = $\frac{Kq_3}{r_3}\hat{r}_3$

En = Electric Field Intensity at a Field Point due to Point Charge qn = $\frac{Kq_n}{r_n}\hat{r}_n$

So we get,

$$E = \frac{Kq_1}{r_2}\hat{r}_1 + \frac{Kq_2}{r_2}\hat{r}_2 + \frac{Kq_3}{r_3}\hat{r}_3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. + \frac{Kq_n}{r_n}\hat{r}_n$$

$$= K\sum_{i=1}^{n}\frac{q_i}{r_i}\hat{r}_i$$

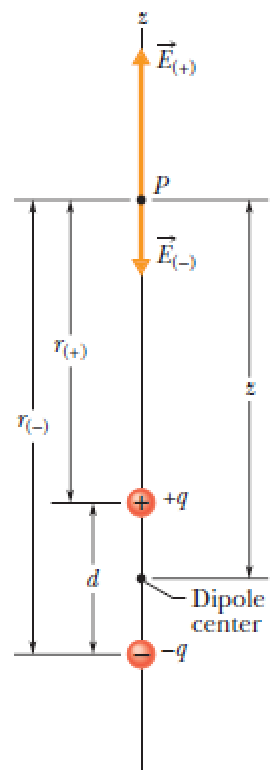
This equation gives the total electric field intensity due to assembly of „n" point charges at a specific field point.

Electric Field due to a Dipole

Two charge particles of magnitude q but of opposite sign, separated by a distance d called electric dipole. Let us find the electric field due to the dipole a point P, a distance z from the midpoint of the dipole and on the axis through the particles, which is called the dipole axis.

The electric field at point P, due to the separate charges are the fields E (+) and E(-) that make up the dipole—must lie along the dipole axis, which we have taken to be a z axis.

The total electric field intensity at point P due to the charges +q and –q is given by the expression.



$$E = E_{(+)} - E_{(-)}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2_{(+)}} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2_{(-)}}$$

$$= \frac{q}{4\pi\varepsilon_0(z - \frac{1}{2}d)^2} - \frac{q}{4\pi\varepsilon_0(z + \frac{1}{2}d)^2}.$$

After forming a common denominator and multiplying its terms, we come to

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\frac{2d/z}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2} = \frac{q}{2\pi\varepsilon_0 z^3}\frac{d}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2}.$$

We are usually interested in the electrical effect of a dipole only at distances that are large compared with the dimensions of the dipole, that is, at distances such that $z >> d$.

At such large distances, $d/2z << 1$

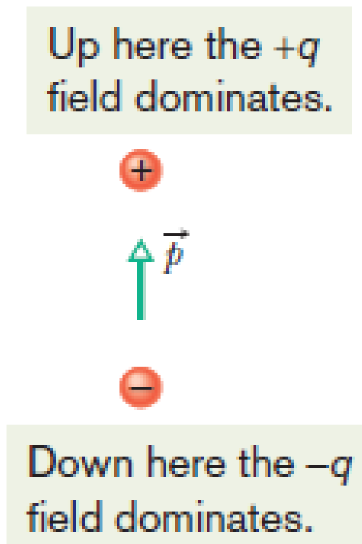
Thus, neglect the $d/2z$ term in the denominator, which leaves us with

$$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}.$$

The product *qd*, known as the electric dipole moment $\vec{p}$ of the dipole, unit of it is the coulomb-meter

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}$$

The direction of dipole moment is taken to be from the negative to the positive end of the dipole. We can use the direction of to specify the orientation of a dipole.



If we measure the electric field of a dipole only at distant points, we can never find q and d separately; instead, we can find only their product.

The field at distant points would be unchanged if, for example, q were doubled and d simultaneously halved.

The direction of E for distant points on the dipole axis is always the direction of the dipole moment vector.
If you double the distance of a point from a dipole, the electric field at the point drops by a factor of 8 ($1/z^3$).

If you double the distance from a single point charge, the electric field drops only by a factor of 4 ($1/r^2$).

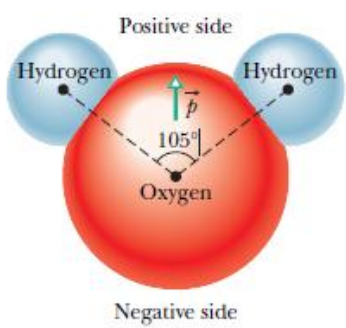
Thus the electric field of a dipole decreases more rapidly with distance than does the electric field of a single charge. The physical reason for this rapid decrease in electric field for a dipole is that from distant points a dipole looks like two equal but opposite charges that almost—but not quite—coincide. Thus, their electric fields at distant points almost—but not quite—cancel each other.

A Dipole in an Electric Field

Electric dipole moment of an electric dipole to be a vector that points from the negative to the positive end of the dipole.

As you will see, the behavior of a dipole in a uniform external electric field can be described completely in terms of the two vectors *E* and *P*, with no need of any details about the dipole's structure.

A molecule of water (H2O) is an electric dipole Fig. There the black dots represent the oxygen nucleus (having eight protons) and the two hydrogen nuclei (having one proton each). The colored enclosed areas represent the regions in which electrons can be located around the nuclei.



In a water molecule, the two hydrogen atoms and the oxygen atom do not lie on a straight line but form an angle of about 105°, as shown in Fig. above. Therefore, the molecule has a definite "oxygen side" and "hydrogen side."

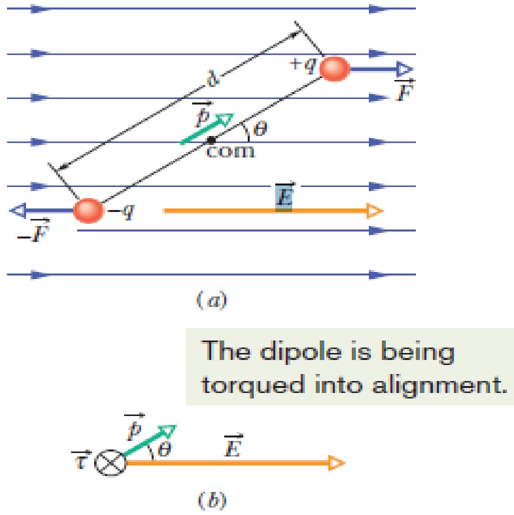
Moreover, the 10 electrons of the molecule tend to remain closer to the oxygen nucleus than to the hydrogen nuclei.

This makes the oxygen side of the molecule slightly more negative than the hydrogen side and creates an electric dipole moment *P* that points along the symmetry axis of the molecule as shown.

If the water molecule is placed in an external electric field, it behaves as would be expected of the more abstract electric dipole of Fig. 22-8.

Now consider such an abstract dipole in a uniform external electric field, as shown in Fig. Assume that the dipole is a rigid structure that consists of two centers of opposite charge, each of magnitude q, separated by a distance d. The dipole moment P makes an angle theta with field E.

a) An electric dipole in a uniform external electric field E:.Two centers of equal but opposite charge are separated by distance d. The line between them represents their rigid connection. (b) Field E: causes a torque t: on the dipole. The direction of t: is into the page, as represented by the symbol



Electrostatic forces act on the charged ends of the dipole. Because the electric field is uniform, those forces act in opposite directions (as shown in above Fig.) and with the same magnitude *F=qE*. Thus, because the field is uniform, the net force on the dipole from the field is zero and the center of mass of the dipole does not move. However, the forces on the charged ends do produce a net torque *T* on the dipole about its center of mass.

The center of mass lies on the line connecting the charged ends, at some distance *x* from one end and thus a distance (*d- x)* from the other end. We know that (*T= rF* sinθ), we can write the magnitude of the net torque *T* as

$$\tau = Fx \sin\theta + F(d - x)\sin\theta = Fd \sin\theta.$$

We can also write the magnitude of torque in terms of the magnitudes of the electric field E and the dipole moment p = qd. Therefore, we substitute qE for F and p/q for d in above Eq. to find the magnitude of *T*,

$$T = pE \sin\theta$$

In vector form $T = p \times E$

The torque acting on the a dipole tends to rotate the dipole into the direction of field , thereby reducing θ. the rotation is clockwise. We can represent a torque that gives rise to a clockwise rotation by including a minus sign with the magnitude of the torque.

$$T = - pE \sin\theta$$

Potential Energy of an Electric Dipole

Potential energy can be associated with the orientation of an electric dipole in an electric field. The dipole has its least potential energy when it is in its equilibrium orientation, which is when its moment *p* is lined up with the field *E* (then $T = P \times E = 0$). It has greater potential energy in all other orientations.

The potential energy of an electric dipole in an external electric field is choose to be zero when the angle is 90°.We then can find the potential energy *U* of the dipole at any other value of θ with (delta *U*= - *W*) by calculating the work W done by the field on the dipole when the dipole is rotated to that value of θ from 90°.With the help of equation ($W = \int T\, d\theta$), We find that the potential energy U at any angle θ is

$$U = -W = -\int_{90^\circ}^{\theta} \tau \, d\theta = \int_{90^\circ}^{\theta} pE \sin\theta \, d\theta.$$

After evaluating the integral

$$U = -pE\cos\theta.$$

We can generalize this equation to vector form as

$$U = -\vec{p} \cdot \vec{E}$$

Potential energy of the dipole is least ($U = -pE$) when $\theta = 0$ (P and E are in the same direction); the potential energy is greatest when $\theta = 180°$ (p and E are in opposite directions).

When a dipole rotates from an initial orientation θ_i to another orientation θ_f, the work done on the dipole by the electric field is

$$W = -\Delta U = -(U_f - U_i),$$

where U_f and U_i are calculated with Eq. ($U = -pE$). If the change in orientation is caused by an applied torque (commonly said to be due to an external agent), then the work Wa done on the dipole by the applied torque is the negative of the work done on the dipole by the field; that is,

$$W_a = -W = (U_f - U_i).$$

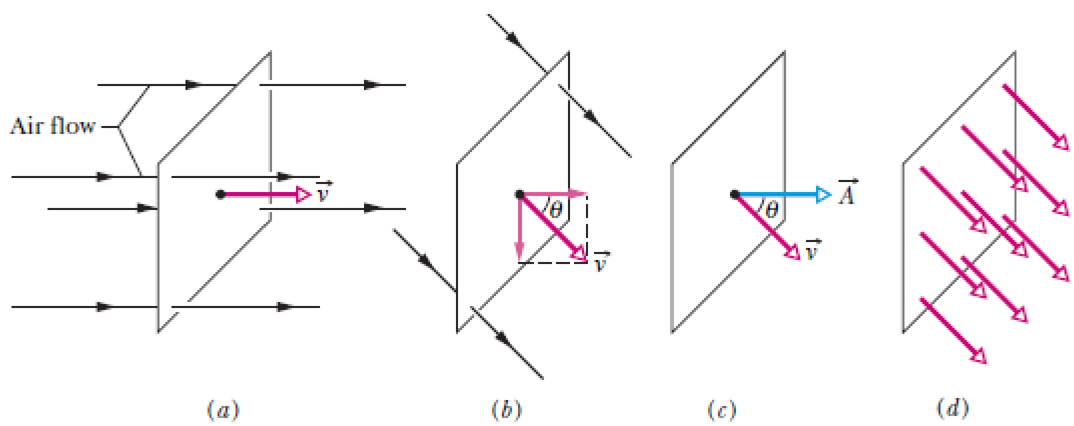
Flux

Suppose that, airstream of uniform velocity *v* passing through at a small square loop of area *A*. Let Φ represent the volume flow rate (volume per unit time) at which air flows through the loop. This rate depends on the angle between v and the plane of the loop. If is perpendicular to the plane, the rate Φ is equal to vA.

If *v* is parallel to the plane of the loop, no air moves through the loop, so Φ is zero. For an intermediate angle θ, the rate Φ depends on the component of *v* normal to the plane. Since that component is v cosθ, the rate of volume flow through the loop is

$$\Phi = (v \cos \theta)A.$$

This rate of flow through an area is an example of a flux—a volume flux in this situation.



(*a*) A uniform airstream of velocity is perpendicular to the plane of a square loop of area *A*.

(*b*) The component of *v* perpendicular to the plane of the loop is *v* cosθ, where θ is the angle between *v* and a normal to the plane.

(*c*) The area vector *A* is perpendicular to the plane of the loop and makes an angle θ with *v*.

(*d*) The velocity field intercepted by the area of the loop.

Area vector as being a vector whose magnitude is equal to an area (here the area of the loop) and whose direction is normal to the plane of the area as shown in fig c.

The scalar (or dot) product of the velocity vector *v* of the airstream and the area vector *A* of the loop

$$\Phi = vA\cos\theta = \vec{v} \cdot \vec{A},$$

Flux of electric field

Gaussian surface immersed in a nonuniform electric field.

Let us divide the surface into small squares of area ΔA, each square being small enough to permit us to neglect any curvature and to consider the individual square to be flat.

ΔA is magnitude of each element of area with an area vector ΔA. Each vector is perpendicular to the Gaussian surface and directed away from the interior of the surface.

Because the squares have been taken to be arbitrarily small, the electric field may be taken as constant over any given square.

The vectors ΔA and E for each square then make some angle θ with each other. Figure shows an enlarged view of three squares on the Gaussian surface and the angle θ for each.

Flux of the electric field for the Gaussian surface

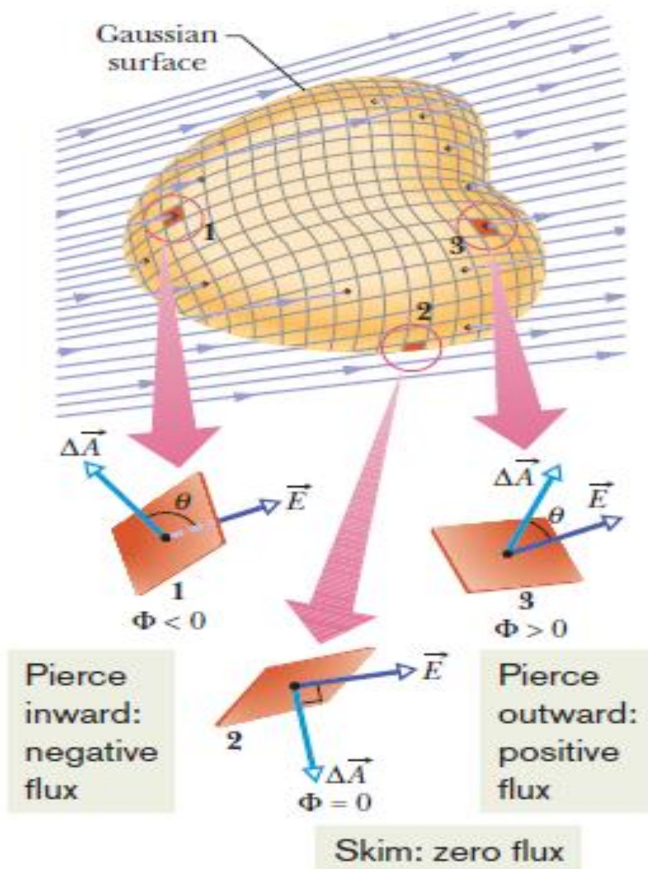
$$\Phi = \sum \vec{E} \cdot \Delta\vec{A}. \qquad (1)$$

For each square on the Gaussian surface, evaluate the scalar product E.ΔA for the two vectors E and ΔA and sum the results algebraically (that is, with signs included) for all the squares that make up the surface. The value of each scalar product (positive, negative, or zero) determines whether the flux through its square is positive, negative, or zero.

Squares like 1 in above Fig., in which points inward, make a negative contribution to the sum of Eq. 1.

Squares like 2, in which E lies in the surface, make zero contribution.

Squares like 3, in which E points outward, make a positive contribution.



The exact definition of the flux of the electric field through a closed surface is found by allowing the area of the squares to become smaller and smaller, approaching a differential limit *dA*. The area vectors then approach a differential limit *dA*. The Eq. 1 becomes an integral:

$$\Phi = \oint \vec{E} \cdot d\vec{A} \qquad \text{(electric flux through a Gaussian surface).}$$

The integration is taken over the entire closed surface. The flux of the electric field is a scalar, and its SI unit is the N.m2/C.

We can interpret the above Eq. in the following way:

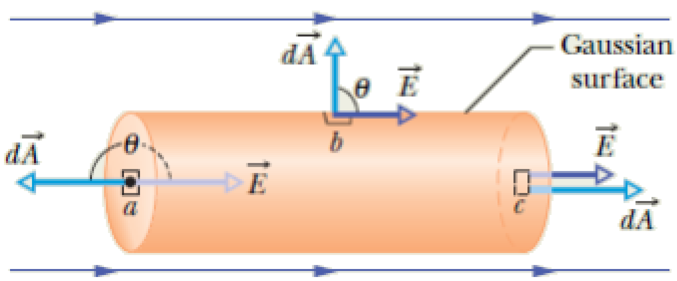
First recall that we can use the density of electric field lines passing through an area as a proportional measure of the magnitude of the electric field there.

Specifically, the magnitude E is proportional to the number of electric field lines per unit area. Thus, the scalar product E. ΔA is proportional to the number of electric field lines passing through area .Then, because the integration in Eq.1 is carried out over a Gaussian surface, which is closed, we see that

The electric flux "through a Gaussian surface is proportional to the net number of electric field lines passing through that surface.

Flux through a closed cylinder, uniform field

A cylinder of radius R immersed in a uniform electric field E, with the cylinder axis parallel to the field. What is the flux of the electric field through this closed surface?



We can do the integration by writing the flux as the sum of three terms: integrals over the left cylinder cap a, the cylindrical surface b, and the right cap c

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

$$= \int_a \vec{E} \cdot d\vec{A} + \int_b \vec{E} \cdot d\vec{A} + \int_c \vec{E} \cdot d\vec{A}.$$

For all points on the left cap, the angle between E and dA is 180° and the magnitude E of the field is uniform. Thus

$$\int_a \vec{E}\cdot d\vec{A} = \int E(\cos 180°)\, dA = -E\int dA = -EA,$$

where ∫dA gives the cap's area A (π R^2). Similarly, for the right cap, where θ = 0 for all points,

$$\int_c \vec{E}\cdot d\vec{A} = \int E(\cos 0)\, dA = EA.$$

Finally, for the cylindrical surface, where the angle is 90° at all points,

$$\int_b \vec{E}\cdot d\vec{A} = \int E(\cos 90°)\, dA = 0.$$

$$\Phi = -EA + 0 + EA = 0.$$

The net flux is zero because the field lines that represent the electric field all pass entirely through the Gaussian surface, from the left to the right.

Gauss' Law

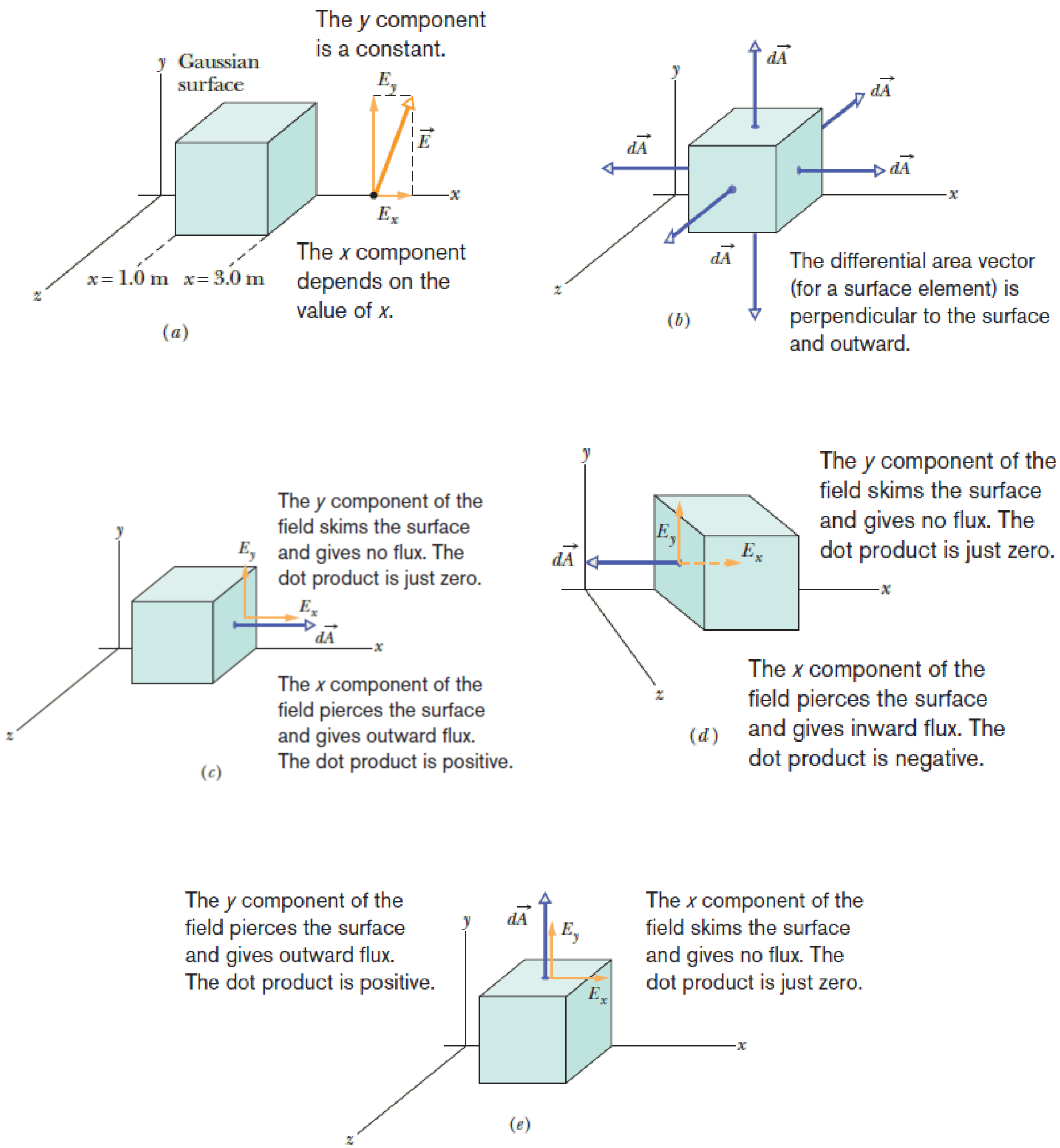
Gauss' law is an expression of the general relationship between the net electric flux through a closed surface and the charge enclosed by the surface. The closed surface is often called a Gaussian surface.

Gauss' law relates the net flux Φ" of an electric field through a closed surface (a Gaussian surface) to the net charge q_{enc} that is enclosed by that surface. It tells us that

$$\varepsilon_0\Phi = q_{\text{enc}} \qquad \text{(Gauss' law)}.$$

By definition of flux, we can also write Gauss' law as

$$\varepsilon_0 \oint \vec{E}\cdot d\vec{A} = q_{\text{enc}} \qquad \text{(Gauss' law)}.$$



(a) A Gaussian cube with one edge on the x axis lies within a nonuniform electric field that depends on the value of x. (b) Each differential area element has an outward vector that is perpendicular to the area. (c) Right face: the x component of the field pierces the area and produces positive (outward) flux. The y component does not pierce the area and thus does not produce any flux. (d) Left face: the x component of the field produces negative (inward) flux. (e) Top face: the y component of the field produces positive (outward) flux.

Equations 1 and 2 hold only when the net charge is located in a vacuum or (what is the same for most practical purposes) in air. In Chapter 25, we modify Gauss' law to include situations in which a material such as mica, oil, or glass is present.

In Eqs. 1 and 2, the net charge q_{enc} is the algebraic sum of all the enclosed positive and negative charges, and it can be positive, negative, or zero. We include the sign, rather than just use the magnitude of the enclosed charge, because the sign tells us something about the net flux through the Gaussian surface:

If q_{enc} is positive, the net flux is outward; if q_{enc} is negative, the net flux is inward. Charge outside the surface, no matter how large or how close it may be, is not included in the term q_{enc} in Gauss' law. The exact form and location of the charges inside the Gaussian surface are also of no concern; the only things that matter on the right side of Eqs. 1 and 2 are the magnitude and sign of the net enclosed charge.

Gauss' Law and Coulomb's Law

To derive Coulomb's law from Gauss law and some symmetry considerations,

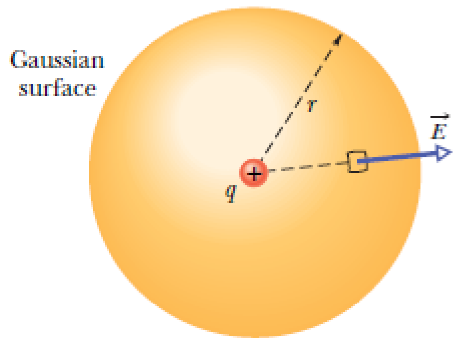


Figure 1 shows a positive point charge q, around which we have drawn a concentric spherical Gaussian surface of radius r. Let us divide this surface into differential areas dA. By definition, the area vector dA at any point is perpendicular to the surface and directed outward from the interior. From the symmetry of the situation, we know that at any point the electric field is also perpendicular to the surface and directed outward from the interior. Thus, since the angle between E and dA is zero, we can rewrite Eq. 2 for Gauss' law as

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 \oint E\, dA = q_{\text{enc}}.$$

Here $qe_{nc} = q$. Although E varies radially with distance from q, it has the same value everywhere on the spherical surface. Since the integral in Eq above is taken over that surface, E is a constant in the integration and can be brought out in front of the integral sign. That gives us

$$\varepsilon_0 E \oint dA = q\,.$$

The integral is now simply the sum of all the differential areas dA on the sphere and thus is just the surface area, $4\pi r^2$. Substituting this, we have

$$\varepsilon_0 E(4\pi r^2) = q$$
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}.$$

The same was found using Coulomb's law.

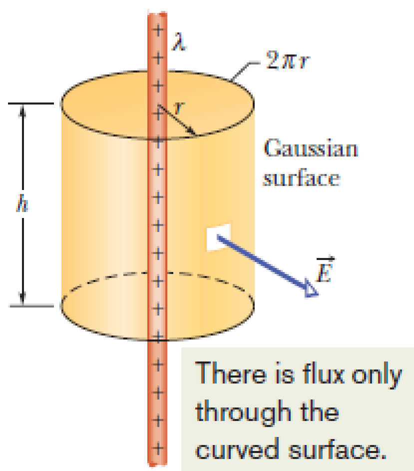
Applying Gauss' Law:

Cylindrical Symmetry

Figure shows a section of an infinitely long cylindrical plastic rod with a uniform positive linear charge density λ. Let us find an expression for the magnitude of the electric field at a distance r from the axis of the rod.

Our Gaussian surface should match the symmetry of the problem, which is cylindrical. We choose a circular cylinder of radius *r* and length *h*, coaxial with the rod. Because the Gaussian surface must be closed, we include two end caps as part of the surface.

Imagine now that, while you are not watching, someone rotates the plastic rod about its longitudinal axis or turns it ends for end. When you look again at the rod, you will not be able to detect any change.



We conclude from this symmetry that the only uniquely specified direction in this problem is along a radial line. Thus, at every point on the cylindrical part of the Gaussian surface, ***E*** must have the same magnitude ***E*** and (for a positively charged rod) must be directed radially outward.

Since $2\pi r$ is the cylinder's circumference and h is its height, the area A of the cylindrical surface is $2\pi rh$.The flux of E through this cylindrical surface is then

$$\Phi = EA\cos\theta = E(2\pi rh)\cos 0 = E(2\pi rh)$$

There is no flux through the end caps because E, being radially directed, is parallel to the end caps at every point.

The charge enclosed by the surface is λh, which means Gauss' law,

$$\varepsilon_0\Phi = q_{\text{enc}},$$
$$\varepsilon_0 E(2\pi rh) = \lambda h,$$
$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

This is the electric field due to an infinitely long, straight line of charge, at a point that is a radial distance r from the line. The direction of E is radially outward from the line of charge if the charge is positive, and radially inward if it is negative. Equation also approximates the field of a finite line of charge at points that are not too near the ends (compared with the distance from the line).

Applying Gauss' Law: Planar Symmetry

Non-conducting Sheet

Figure shows a portion of a thin, infinite, non-conducting sheet with a uniform (positive) surface charge density σ. A sheet of thin plastic wrap, uniformly charged on one side, can serve as a simple model. Let us find the electric field a distance r in front of the sheet.

A useful Gaussian surface is a closed cylinder with end caps of area A, arranged to pierce the sheet perpendicularly as shown. From symmetry, E must be perpendicular to the sheet and hence to the end caps. Furthermore, since the charge is positive, ***E*** is directed away from the sheet, and thus the electric field lines pierce the two Gaussian end caps in an outward direction. Because the field lines do not pierce the curved surface, there is no flux through this portion of the Gaussian surface. Thus E dA is simply E dA; then Gauss' law,

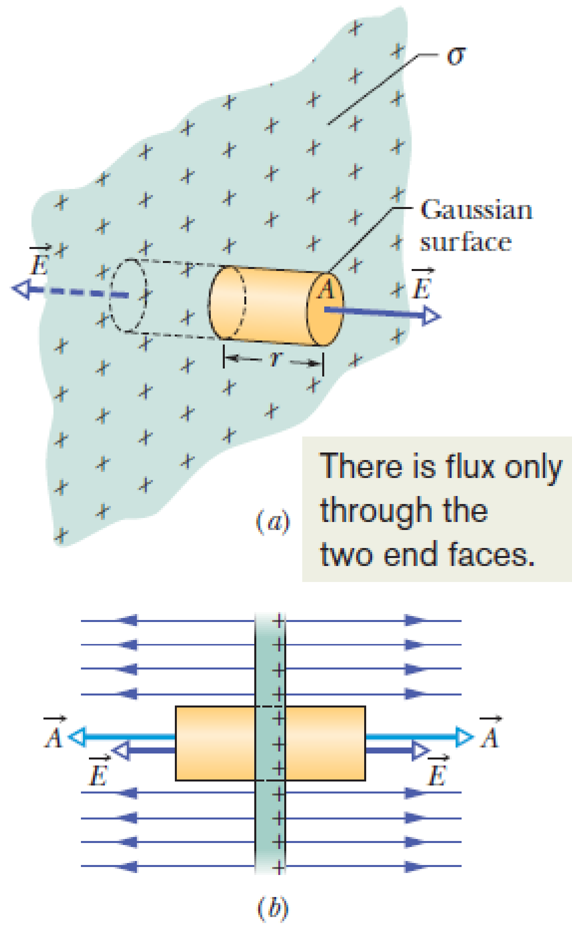
$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}},$$

$$\varepsilon_0(EA + EA) = \sigma A,$$

where σA is the charge enclosed by the Gaussian surface. This gives

$$E = \frac{\sigma}{2\varepsilon_0}$$

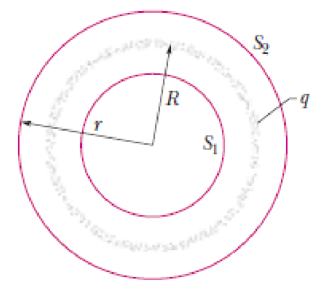
Since we are considering an infinite sheet with uniform charge density, this result holds for any point at a finite distance from the sheet.



(a) Perspective view and (b) side view of a portion of a very large, thin plastic sheet, uniformly charged on one side to surface charge density σ. A closed cylindrical Gaussian surface passes through the sheet and is perpendicular to it.

Applying Gauss' Law: Spherical Symmetry

Figure shows a charged spherical shell of total charge q and radius R and two concentric spherical Gaussian surfaces, S_1 and S_2. If we applied Gauss' law to surface S_2, for which $r \geq R$, we would find that



$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad (1)$$

A thin, uniformly charged, spherical shell with total charge q, in cross section. Two Gaussian surfaces S_1 and S_2 are also shown in cross section. Surface S_2 encloses the shell, and S_1 encloses only the empty interior of the shell.

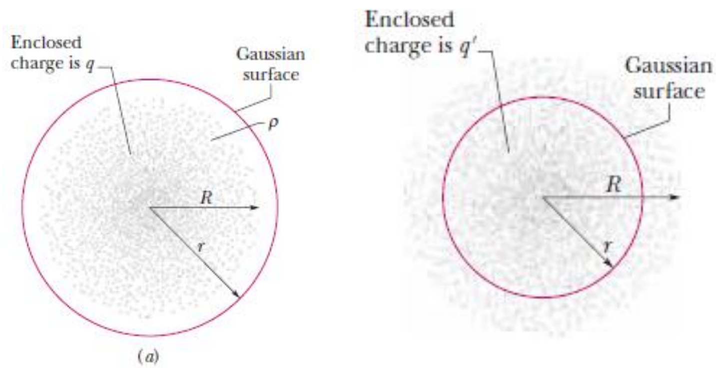
This field is the same as one set up by a point charge q at the center of the shell of charge. Thus, the force produced by a shell of charge q on a charged particle placed outside the shell is the same as the force produced by a point charge q located at the center of the shell. This proves the first shell theorem.

Applying Gauss' law to surface S_1, for which $r < R$, leads directly to

$$E = 0 \qquad (2)$$

Because this Gaussian surface encloses no charges. Thus, if a charged particle were enclosed by the shell, the shell would exert no net electrostatic force on the particle. This proves the second shell theorem

Any spherically symmetric charge distribution, such as that of Fig. 23-19, can be constructed with a nest of concentric spherical shells. For purposes of applying the two shell theorems, the volume charge density ρ should have a single value for each shell but need not be the same from shell to shell. Thus, for the charge distribution as a whole, r can vary, but only with r, the radial distance from the center. We can then examine the effect of the charge distribution "shell by shell."



In Fig. 23-19a, the entire charge lies within a Gaussian surface with r > R. The charge produces an electric field on the Gaussian surface as if the charge were a point charge located at the center, and Eq. 1 holds.

Figure 23-19b shows a Gaussian surface with r < R. To find the electric field at points on this Gaussian surface, we consider two sets of charged shells—one set inside the Gaussian surface and one set outside. Equation 2 says that the charge lying outside the Gaussian surface does not set up a net electric field on the Gaussian surface.

Equation 1 says that the charge enclosed by the surface sets up an electric field as if that enclosed charge were concentrated at the center. Letting q′ represent that enclosed charge, we can then rewrite Eq. 1 as

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q'}{r^2} \tag{3}$$

If the full charge q enclosed within radius R is uniform, then q′ enclosed within radius r in Fig. b is proportional to q:

$$\frac{\begin{pmatrix}\text{charge enclosed by}\\ \text{sphere of radius } r\end{pmatrix}}{\begin{pmatrix}\text{volume enclosed by}\\ \text{sphere of radius } r\end{pmatrix}} = \frac{\text{full charge}}{\text{full volume}}$$

$$\frac{q'}{\frac{4}{3}\pi r^3} = \frac{q}{\frac{4}{3}\pi R^3}. \tag{4}$$

$$q' = q\frac{r^3}{R^3}. \tag{5}$$

Put this value in eq. 3

$$E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right)r \tag{6}$$